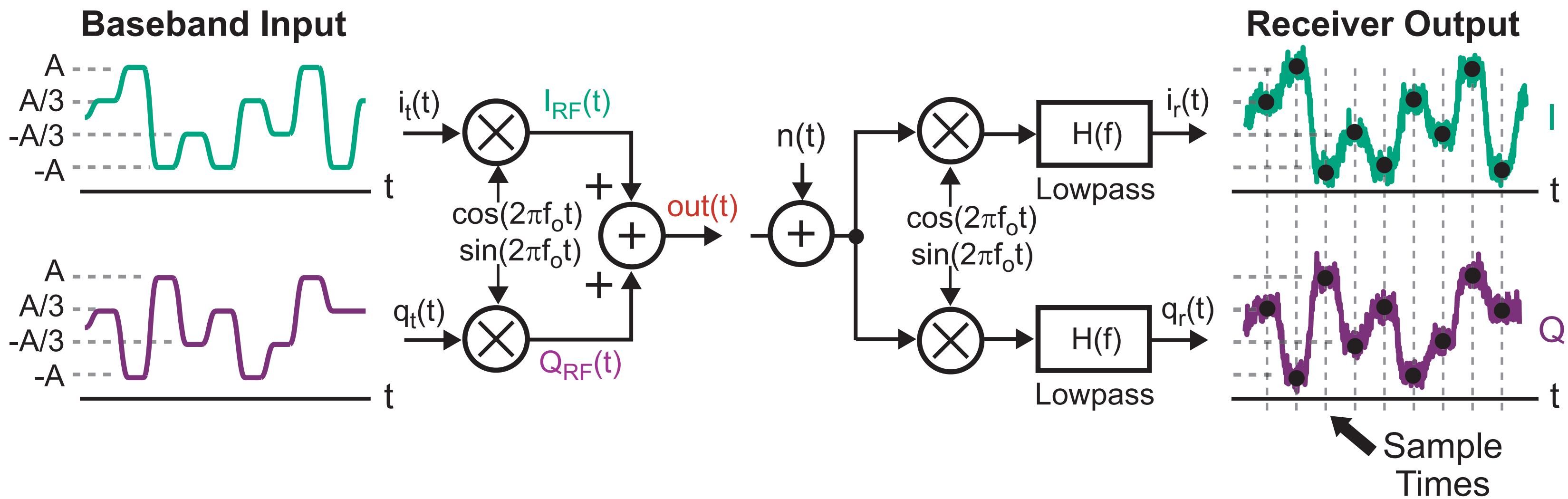
Energy and Noise Revisited

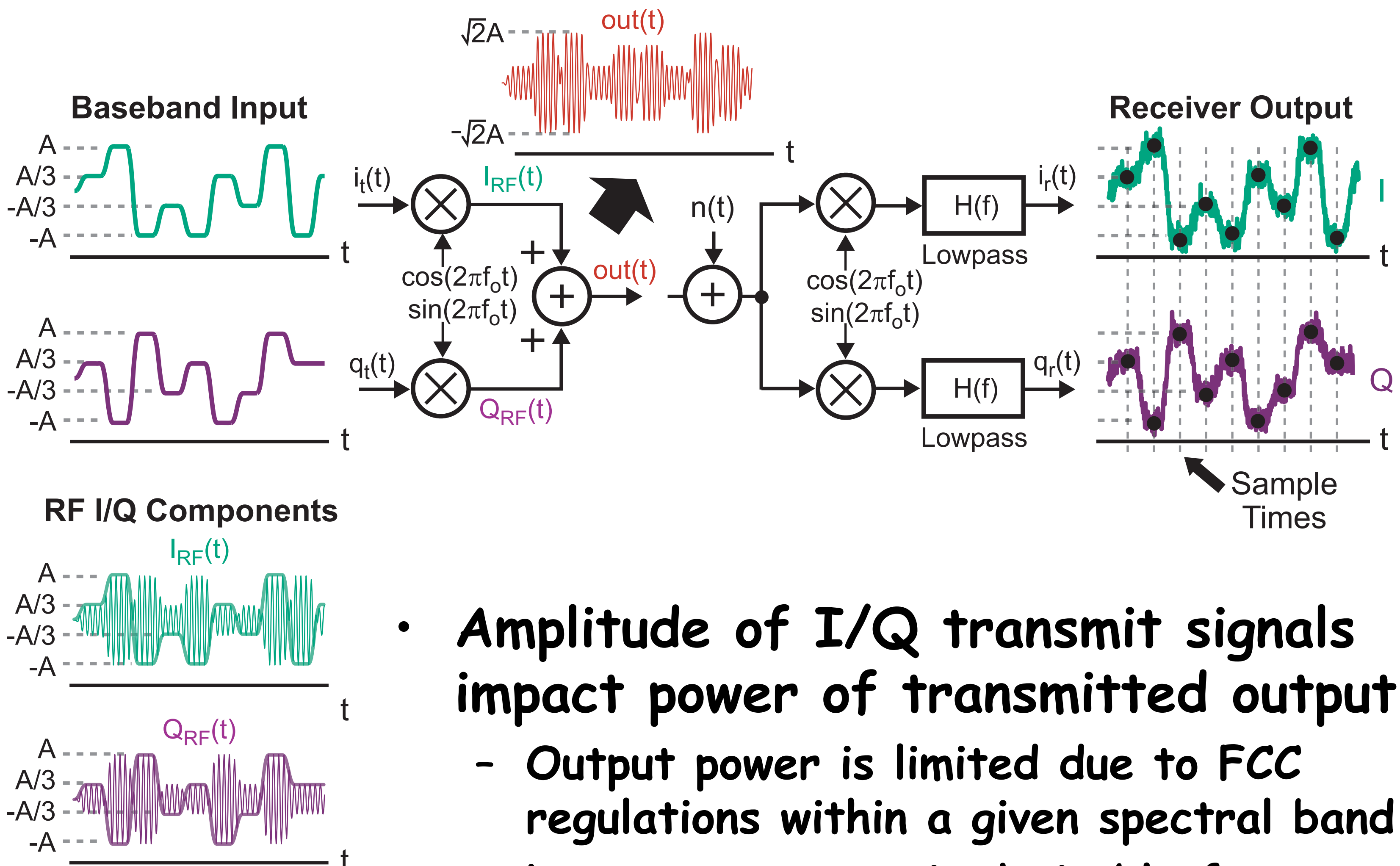
- Constellation diagrams and SNR
- Bit error rate versus SNR
- Shannon Capacity Limit

Review of Digital Modulation



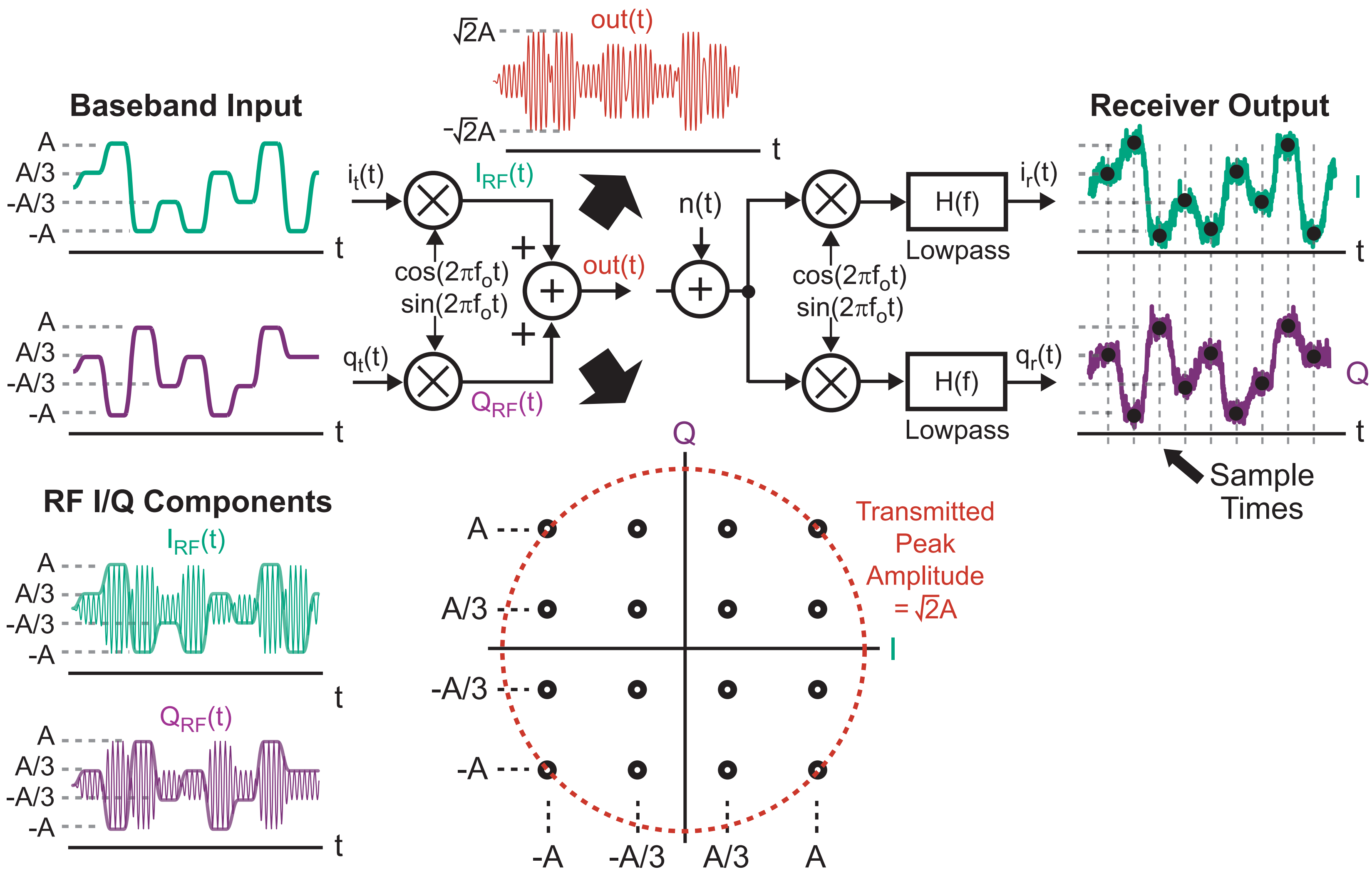
- Transmitter sends discrete-valued signals over an analog communication channel
- Receiver samples recovered baseband signal
  - Noise and ISI corrupt received signal
- Key techniques
  - Properly design transmit and receive filters for low ISI
  - Sample and slice received signals to detect symbols

A Closer Look at the Transmitter



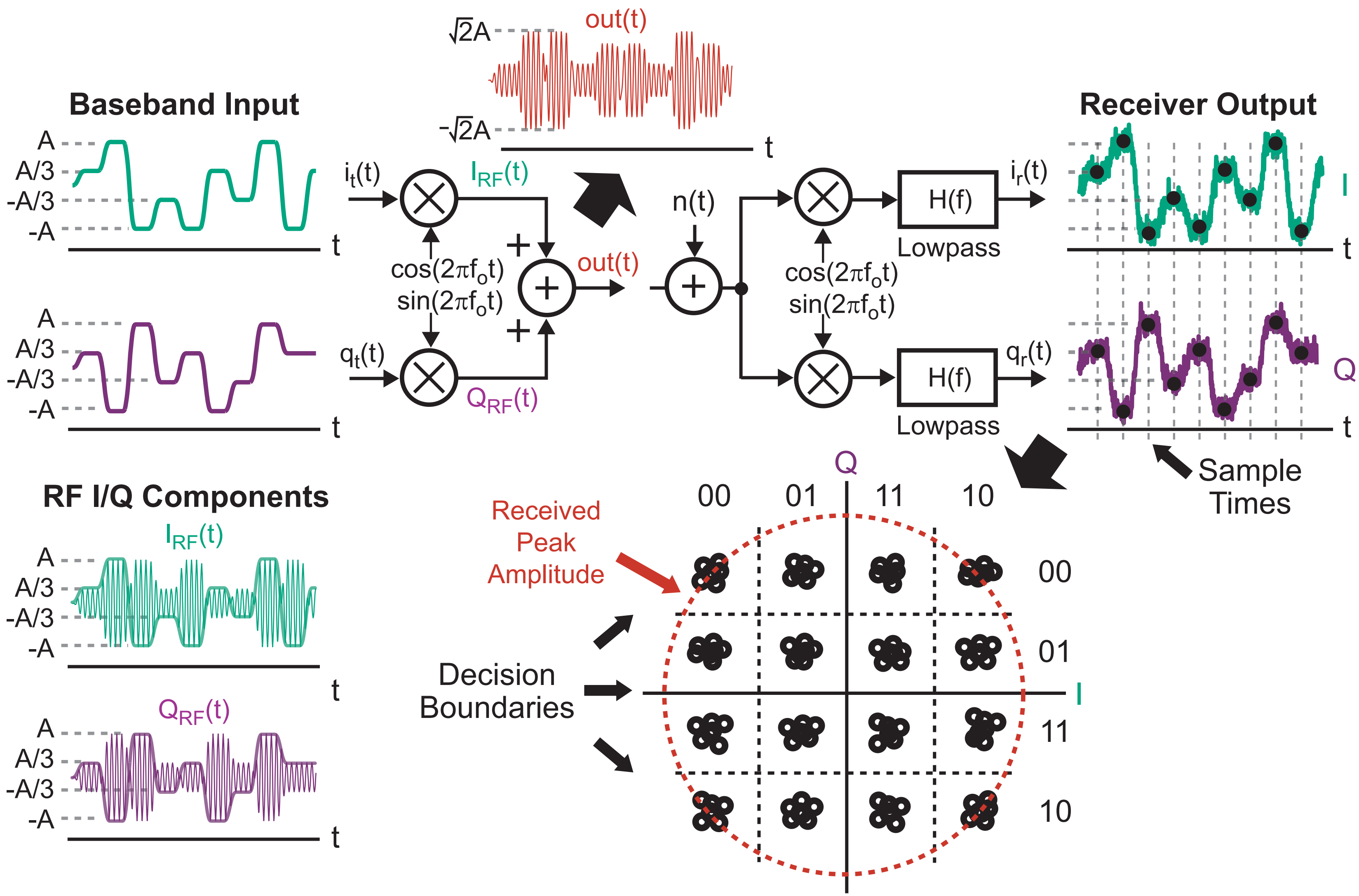
- **Amplitude of I/Q transmit signals impact power of transmitted output**
  - Output power is limited due to FCC regulations within a given spectral band
  - Low output power is desirable for portable applications to achieve long battery life

A Constellation View of Transmitter



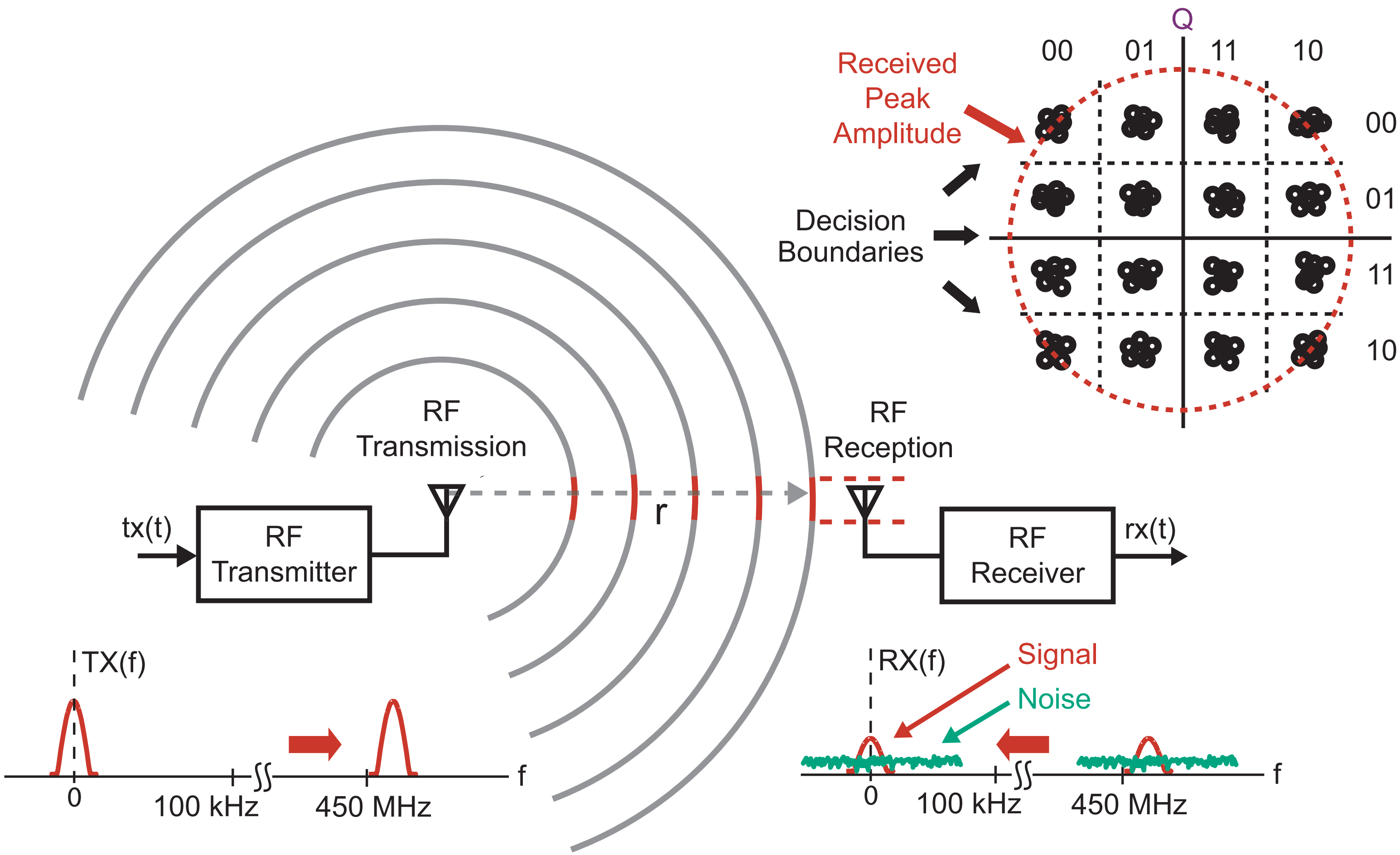
- Provides intuitive view of relationship between symbol separation and transmitted power

A Constellation View of Receiver



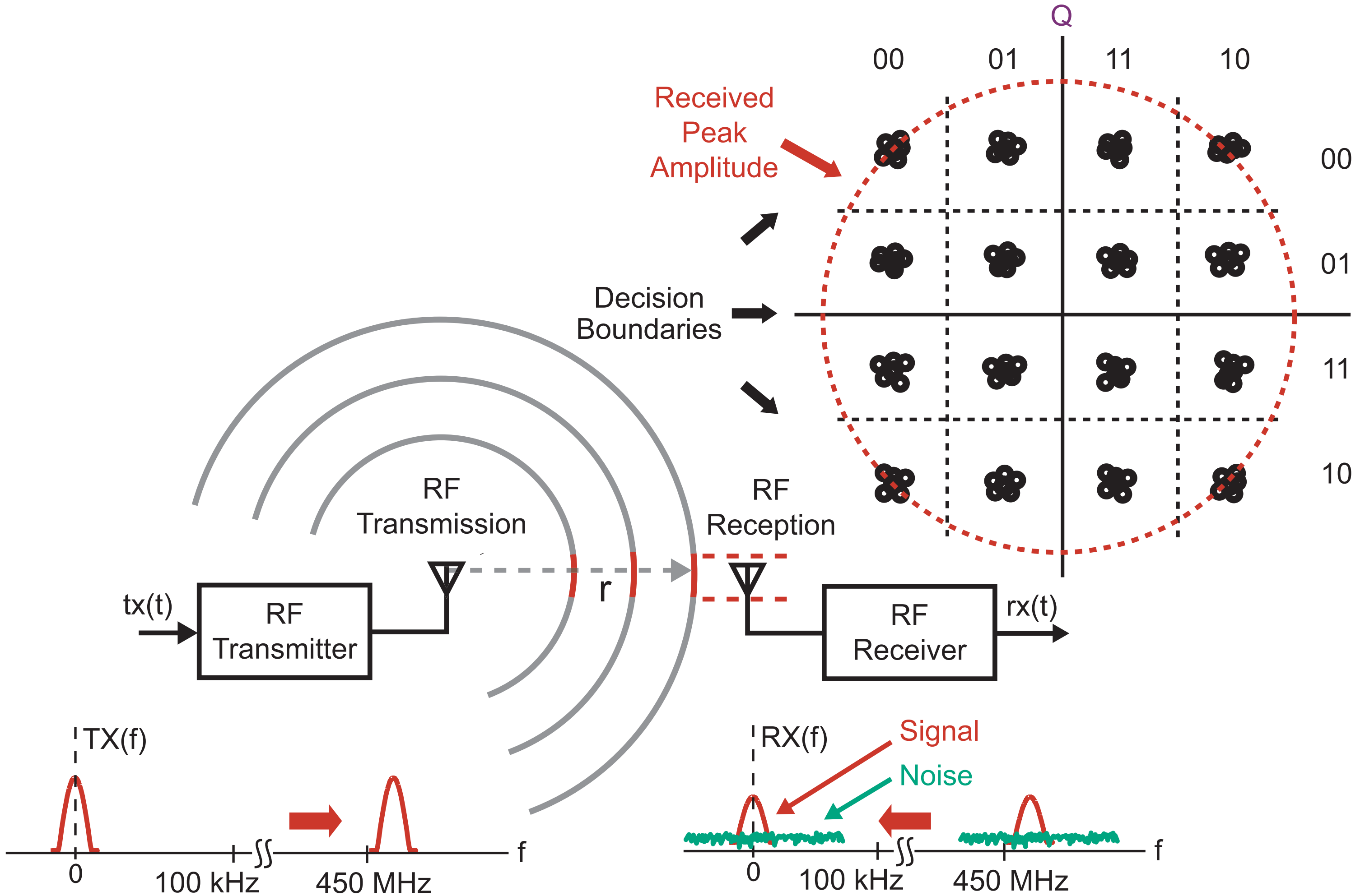
- Provides an intuitive view of relationship between symbol separation, received signal power, and noise

Impact of SNR on Receiver Constellation



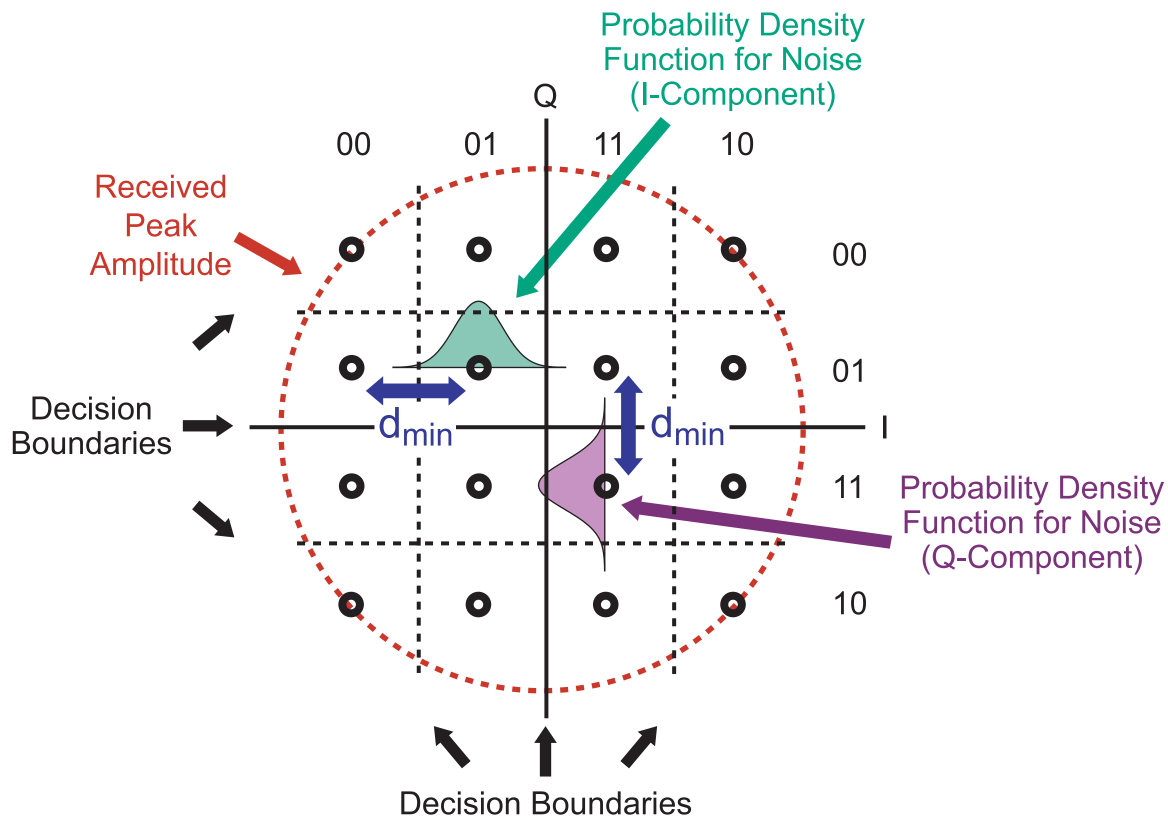
- **SNR influenced by transmitted power, distance between transmitter and receiver, and noise**

Impact of Increased Signal on Constellation



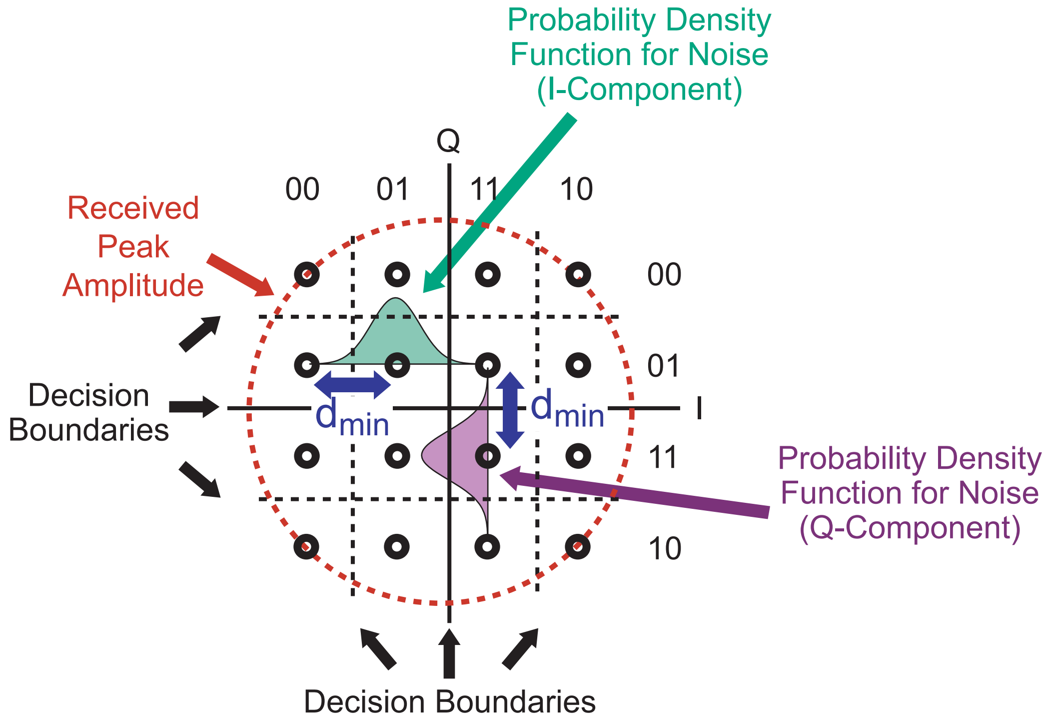
- **Increase in received signal power leads to increased separation between symbols**
  - SNR is improved if noise level unchanged

Quantifying the Impact of Noise



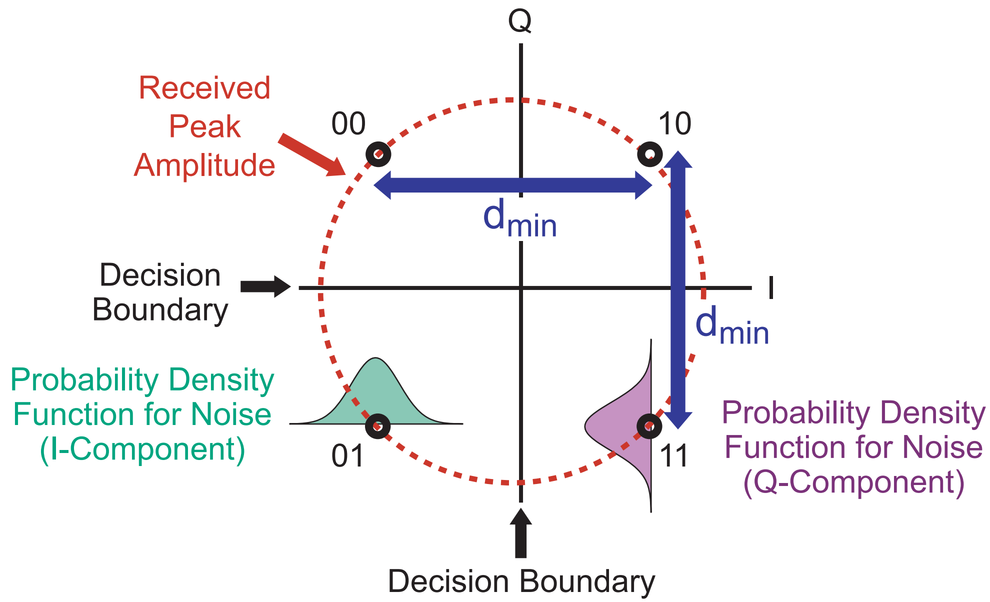
- Minimum separation between symbols: d_{min}
- PDF of noise: zero mean Gaussian PDF
  - Variance of noise sets the spread of the PDF
- Bit errors: occur when noise moves a symbol by a distance more than $d_{min}/2$

Impact of Reduced SNR



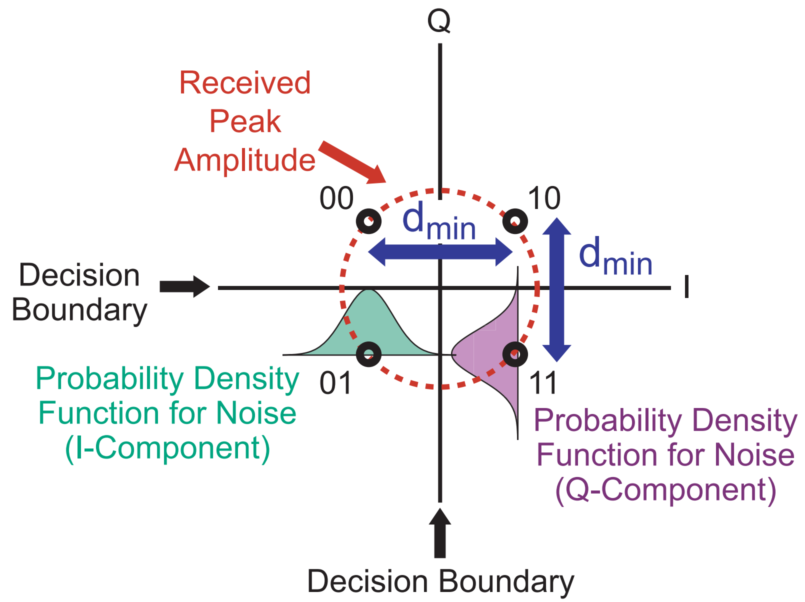
- Lower SNR leads to a reduced value for d_{min}
- Leads to a higher bit error rate
  - Assumes noise variance is unchanged

Impact of Symbol Reduction



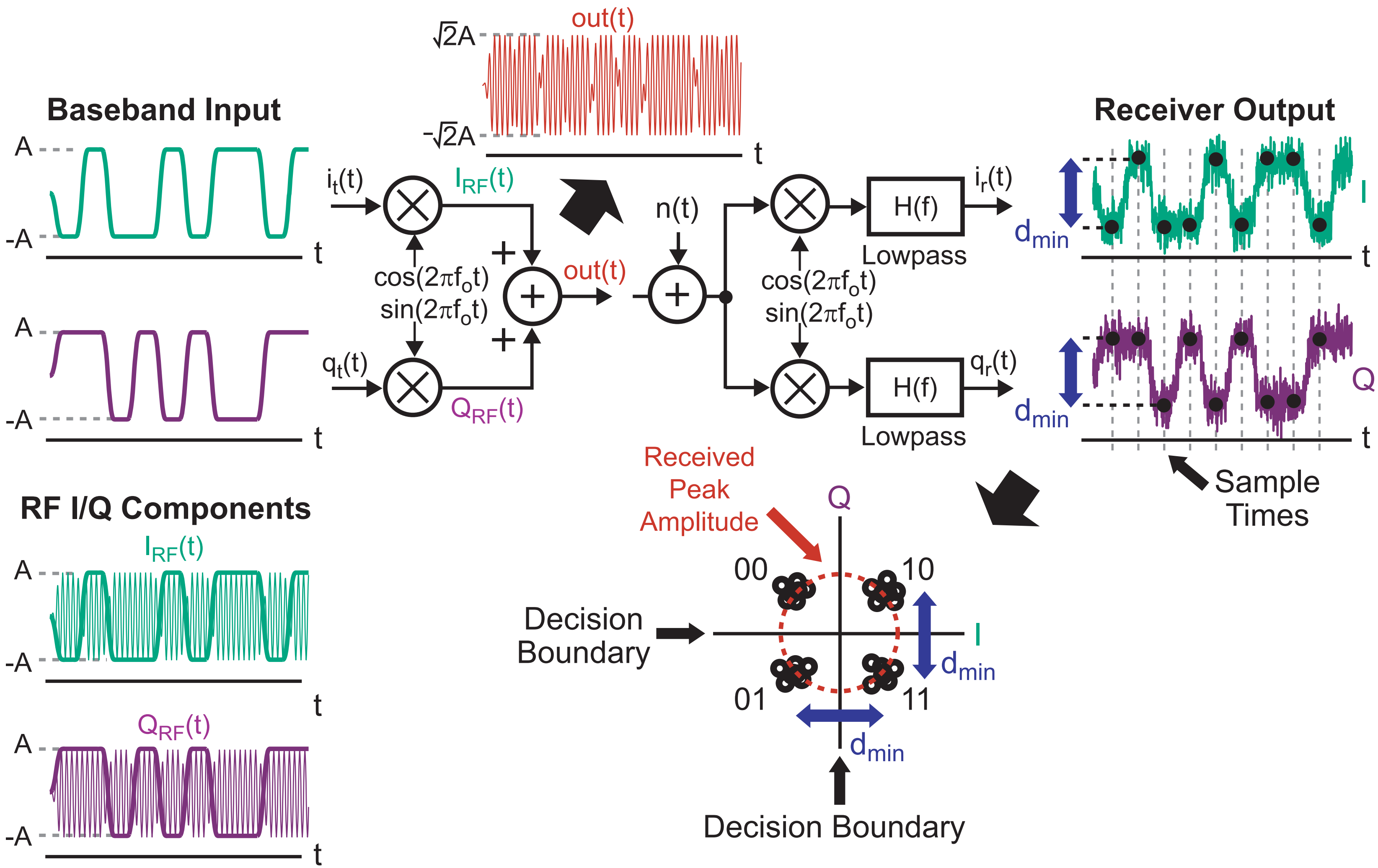
- **Reducing the number of symbols leads to an increased value for d_{min}**
- **Leads to a lower bit error rate**
  - **Assuming SNR remains constant**

Can We Estimate Bit Error Rate?



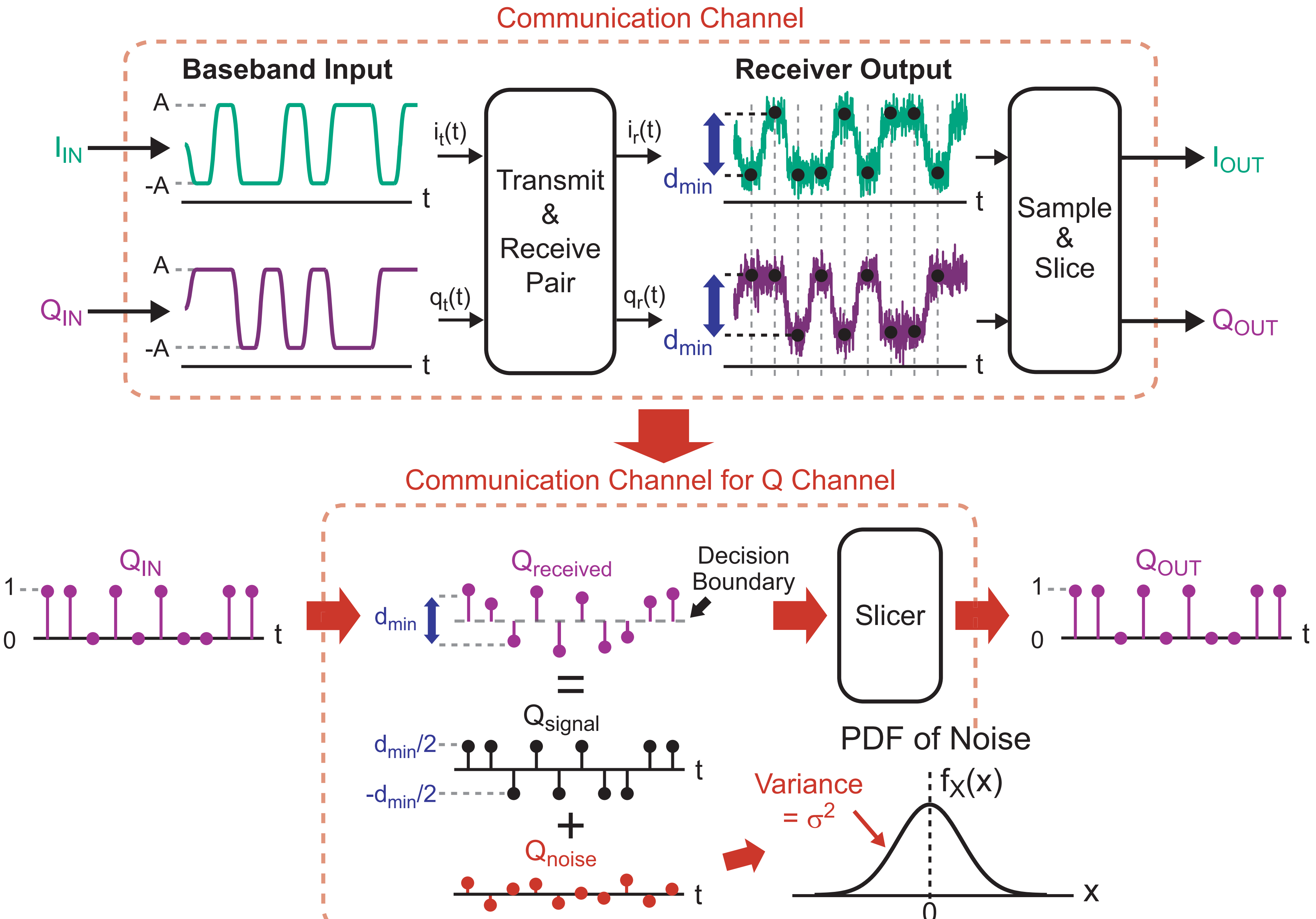
- **Bit Error Rate depends on:**
  - SNR
    - Received signal power versus noise variance
  - Number of constellation points
    - Sets d_{min} at a given level of received signal power

Let's Start with a Detailed System View

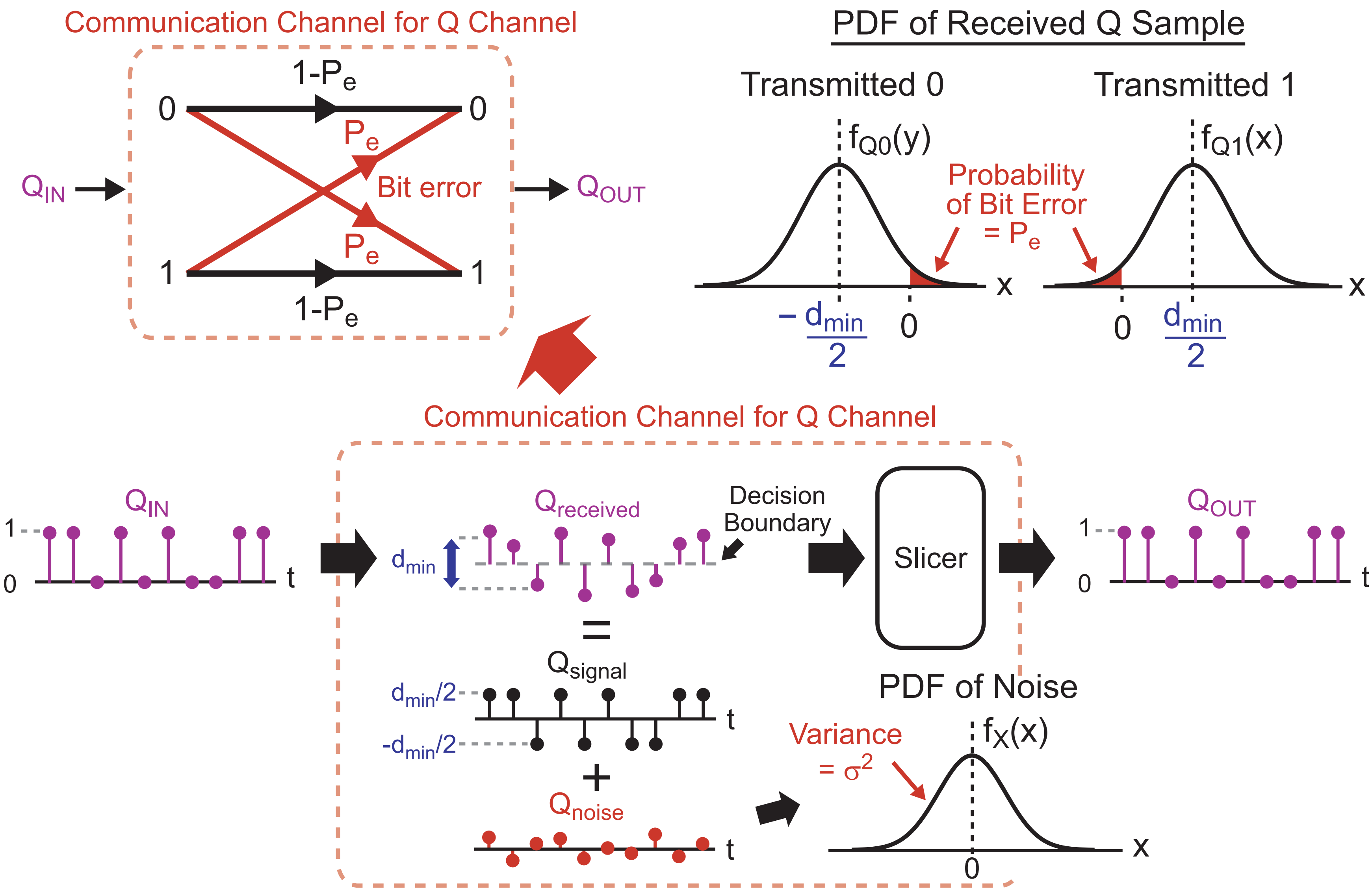


- **Assumptions: No ISI, 4-point constellation**

A Closer Examination of Signal and Noise

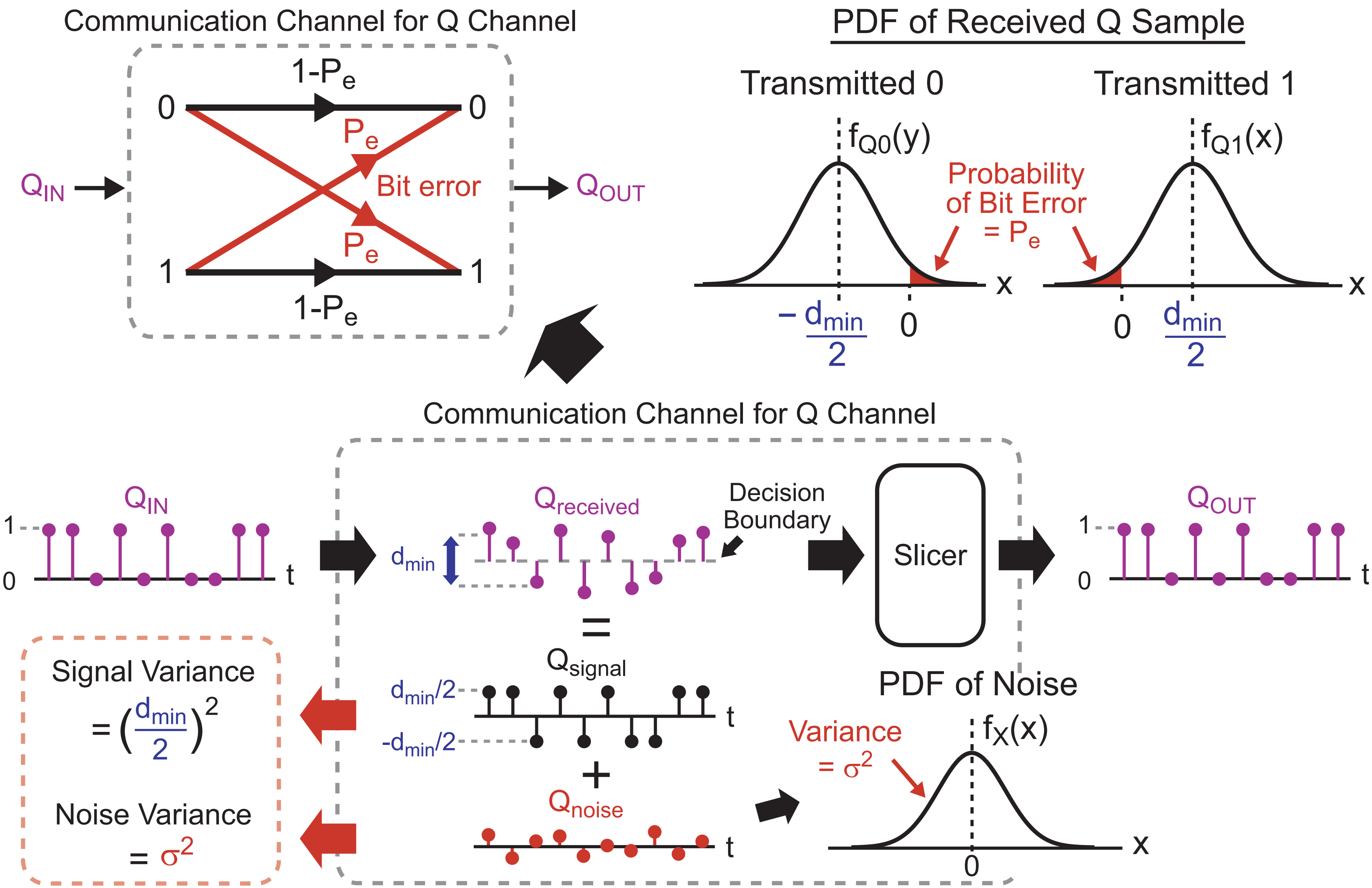


The Binary Symmetric Channel Model



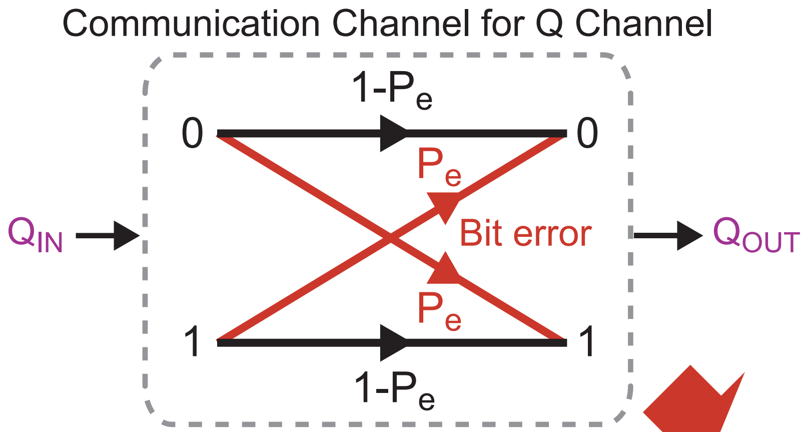
- **Provides a binary signaling model of channel**

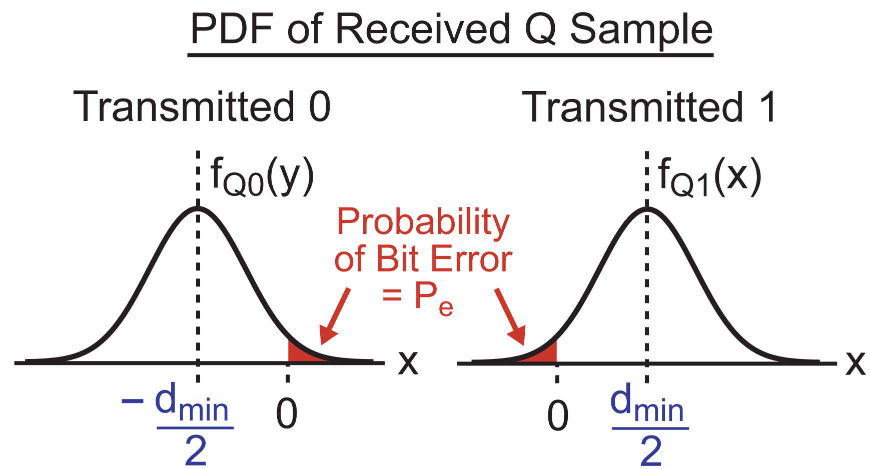
Computation of SNR

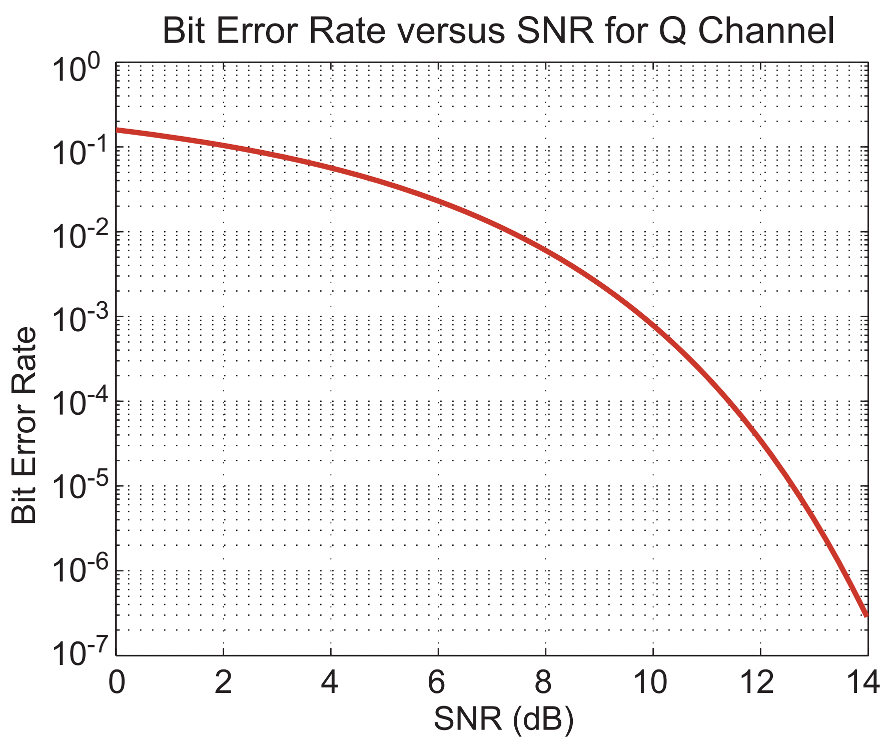


$$\Rightarrow \quad SNR(dB) = 10\log\left((d_{min}/2)^2/\sigma^2\right)$$

Resulting Bit Error Rate Versus SNR







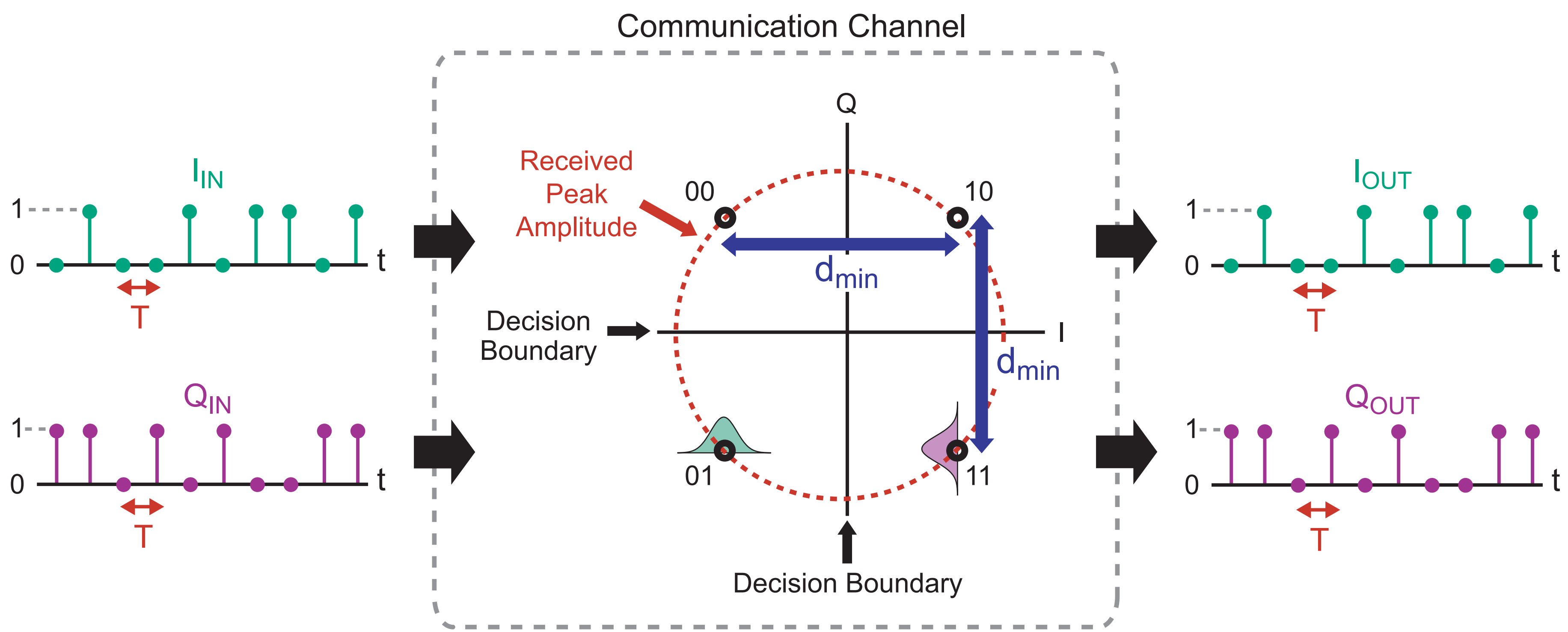
Note:

- **Bit Error Rate = P_e**
- **SNR (dB) =**

$$10 \log\left(\frac{(d_{min}/2)^2}{\sigma^2}\right)$$

- **Gaussian PDF for noise**

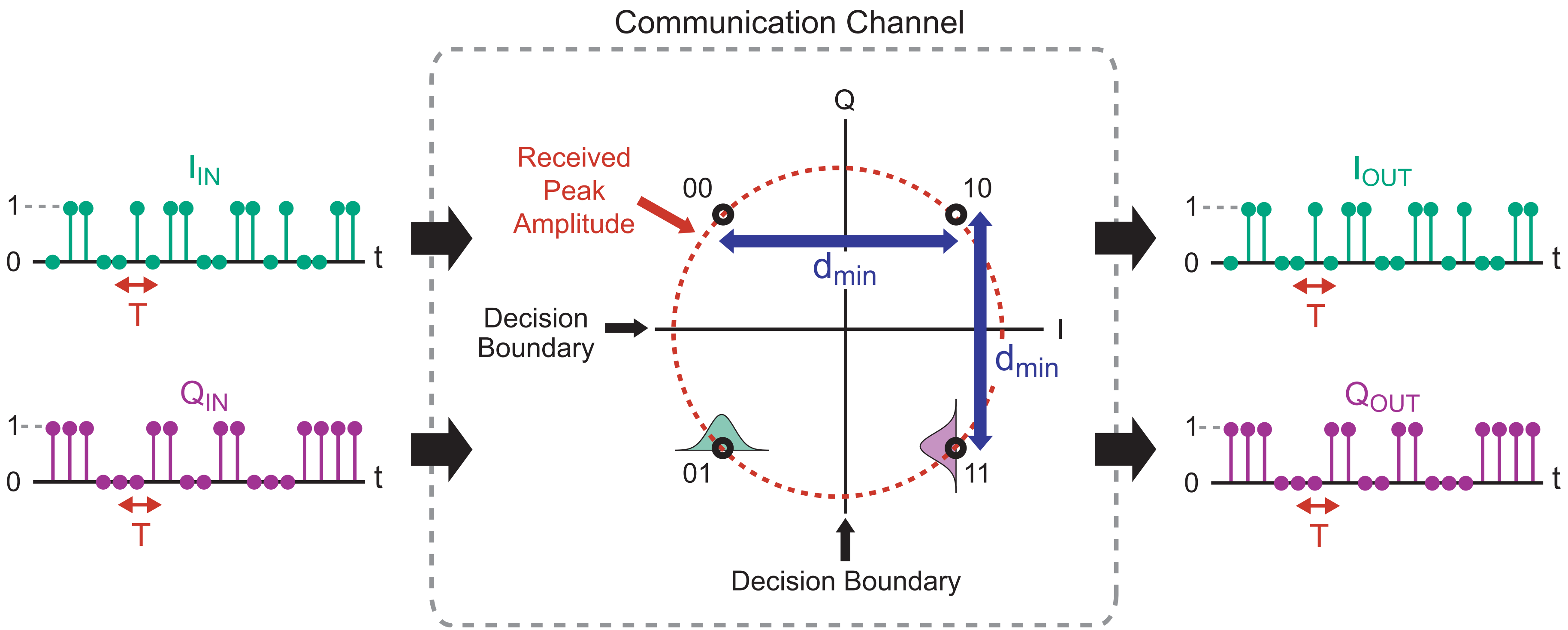
Shannon Capacity



- In 1948, Claude Shannon proved that
  - Digital communication can achieve arbitrary low bit-error-rates if appropriate *coding* methods are employed
  - The capacity of a *Gaussian channel* with bandwidth *BW* to support arbitrary low bit-error-rate communication is:

$$C = BW \log_2(1 + SNR) \text{ bits/second}$$

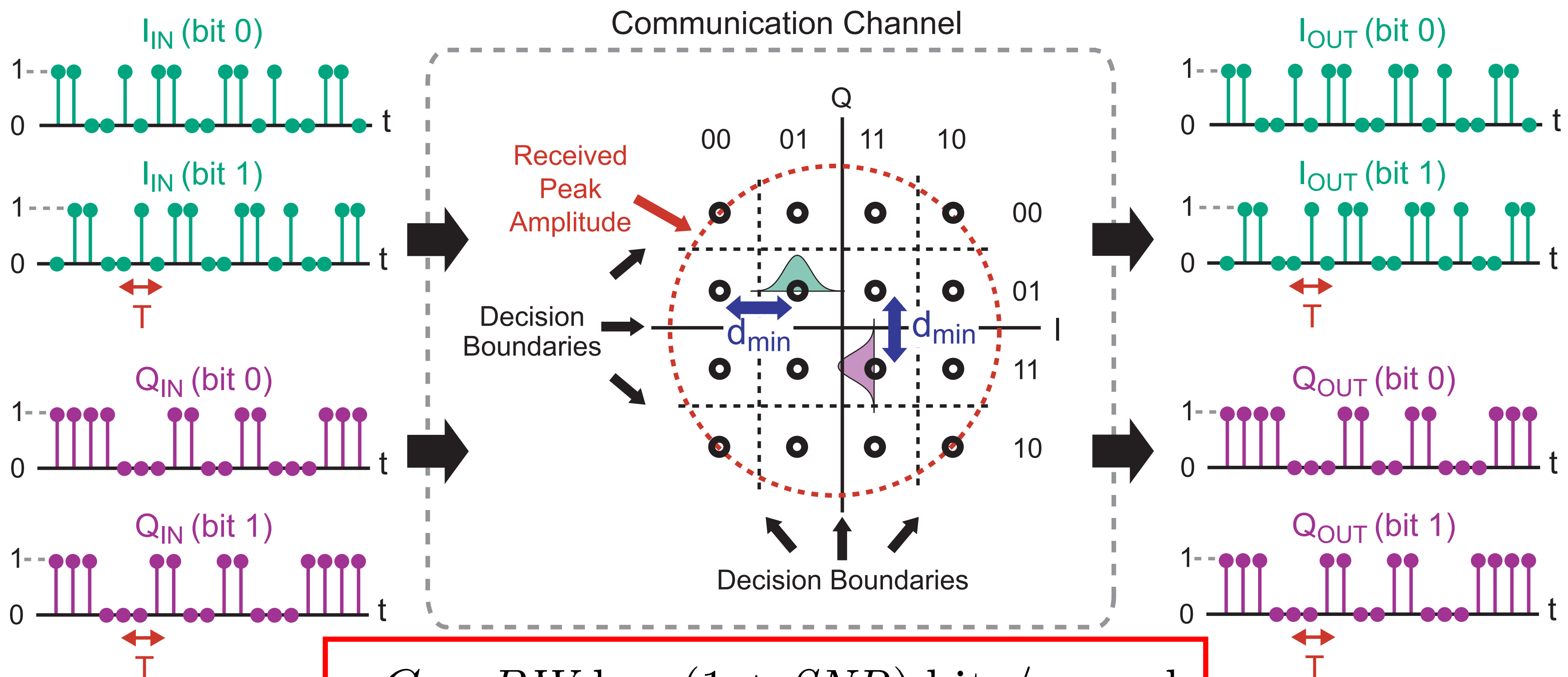
Impact of Channel Bandwidth on Capacity



$$C = BW \log_2(1 + SNR) \text{ bits/second}$$

- **An increase in bandwidth by a factor of 2 allows twice the number of bits to be sent in time T**
  - **Capacity (bits/second) increases *linearly* with bandwidth**

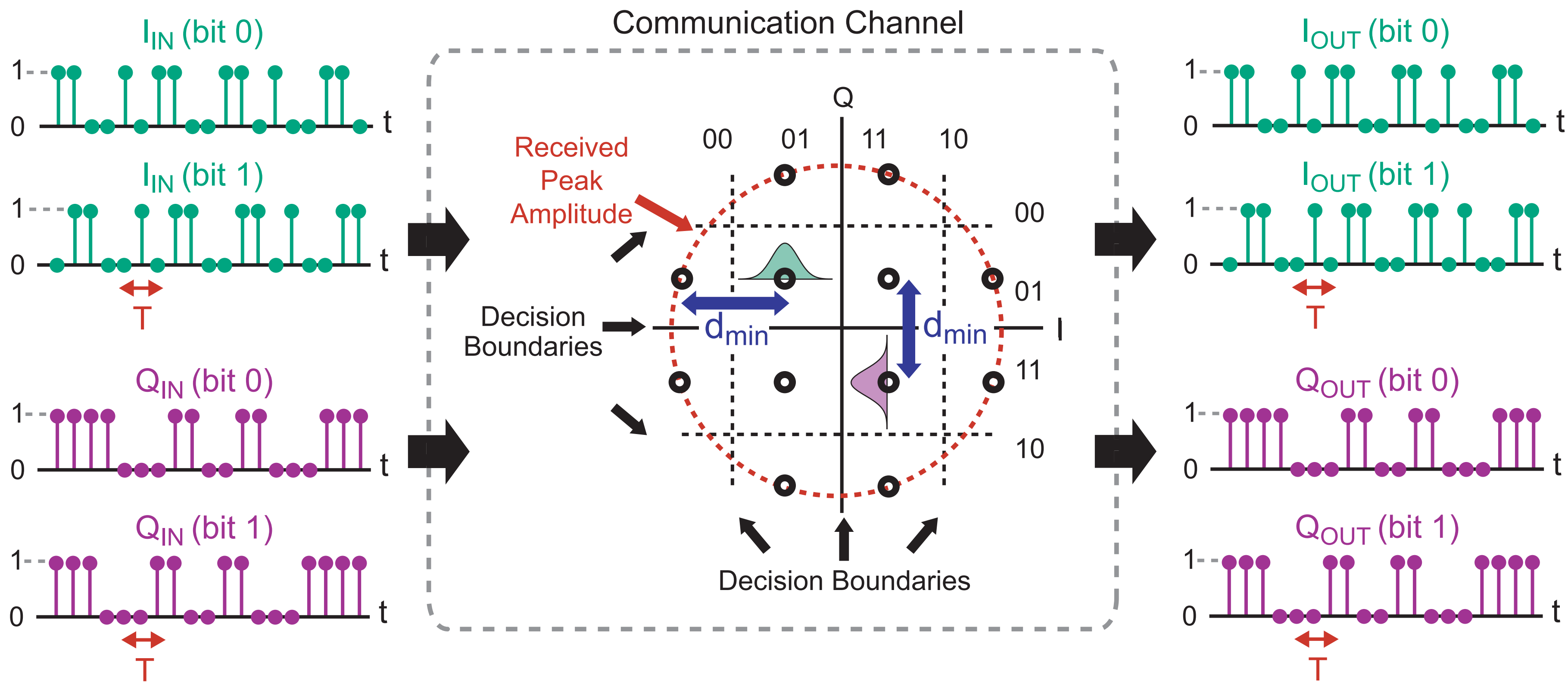
Impact of SNR on Capacity



$$C = BW \log_2(1 + SNR) \text{ bits/second}$$

- A high SNR allows more bits to be sent per symbol
  - Adding *n* bits requires adding 2^n constellation points
    - Adding *n* bits therefore leads to d_{min} being *reduced* by a factor of 2^n
  - Capacity increases *logarithmically* with SNR

Constellation Design (Symbol Packing)



- **Objective: design constellation to maximize d_{min} while packing as many points in as possible**
  - Maximizing d_{min} achieves lowest *uncoded* bit error rate
  - Maximizing number of constellation points achieves highest *uncoded* data rate (bits/second)

Summary

- **Constellation diagrams allow intuitive approach of quantifying *uncoded* bit error rate of a channel**
  - **Function of SNR and number of constellation points**
- **A digital communication channel can be viewed in terms of a binary signaling model**
  - **Focuses attention on key issue of bit error rate**
- **Coding theoretically allows arbitrary low bit-error-rate performance of a practical digital communication link**
  - **We will dive more into this topic in the coming weeks….**
- **Next lecture: Wrap Up**