Short Course On Phase-Locked Loops and Their Applications Day 2, AM Lecture

Basic Building Blocks Voltage-Controlled Oscillators

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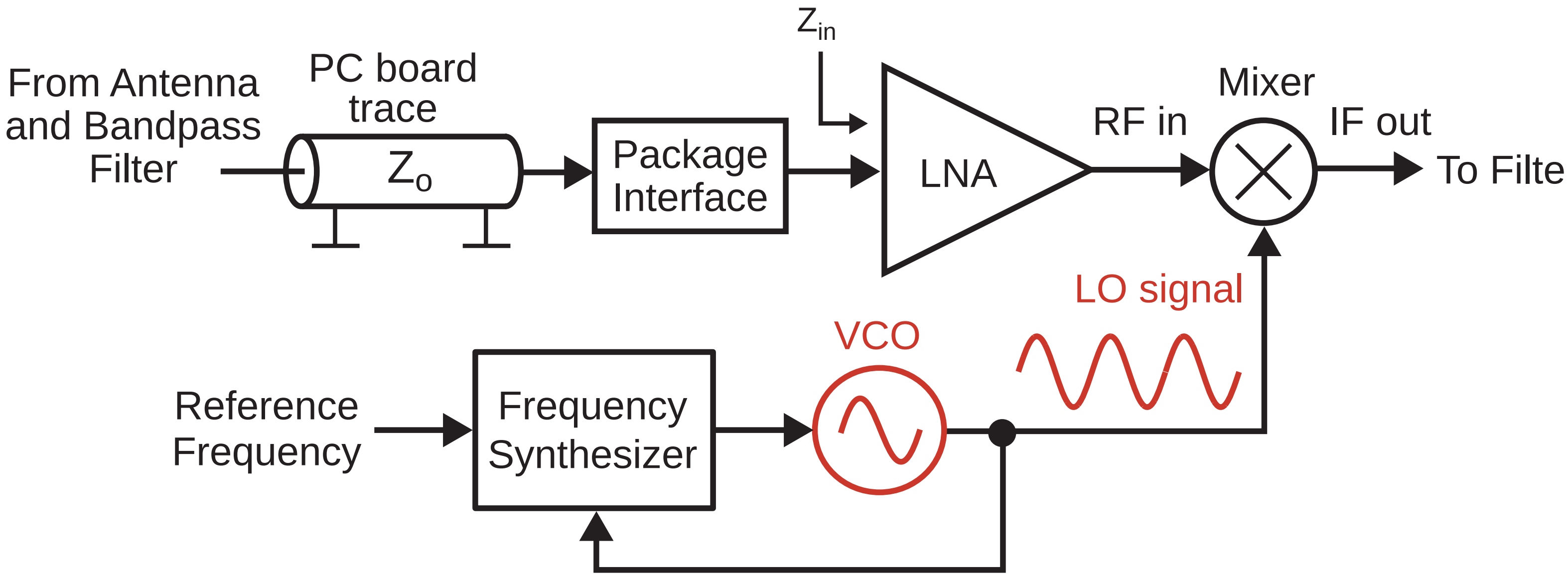
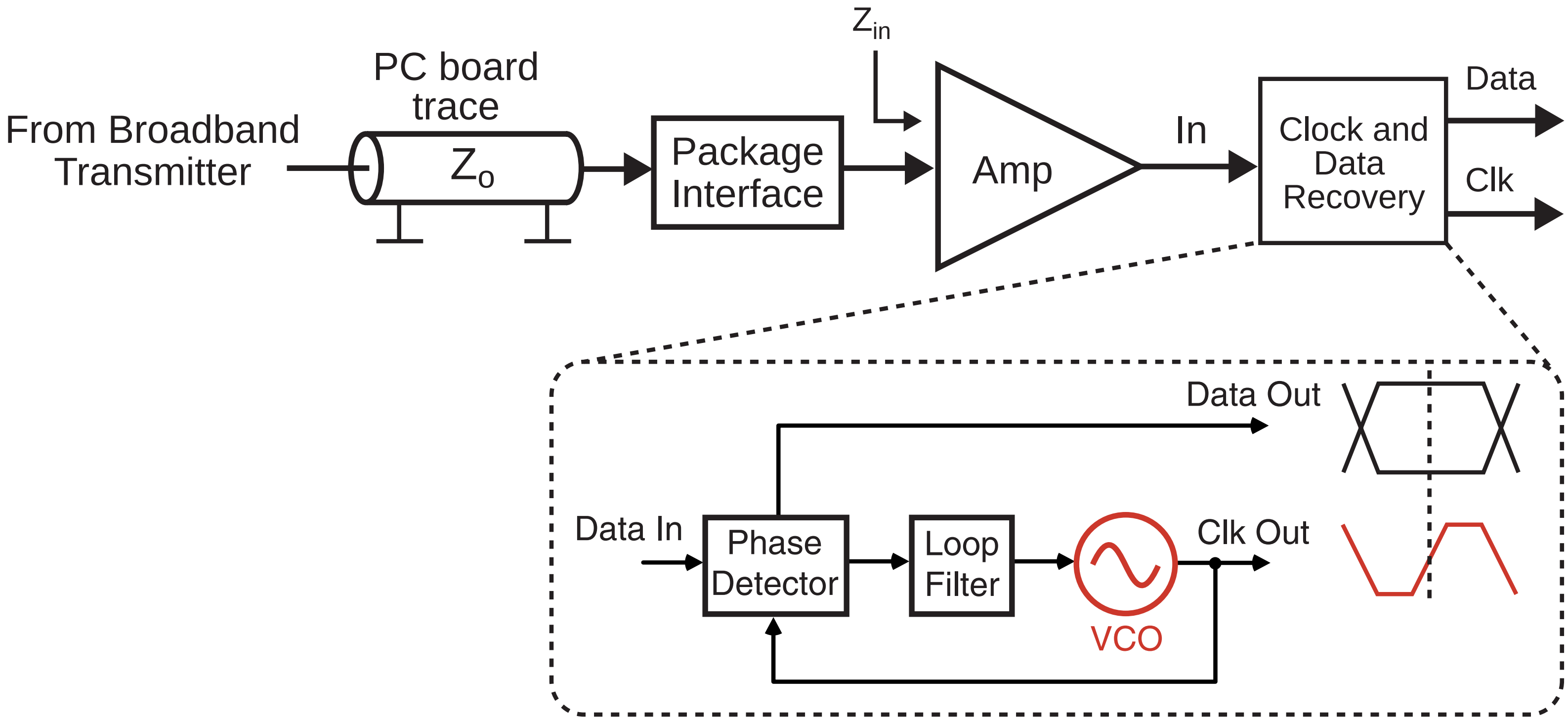
VCO Design for Wireless Systems



- **Design Issues**
  - **Tuning Range – need to cover all frequency channels**
  - **Noise – impacts receiver blocking and sensitivity performance**
  - **Power – want low power dissipation**
  - **Isolation – want to minimize noise pathways into VCO**
  - **Sensitivity to process/temp variations – need to make it manufacturable in high volume**

VCO Design For High Speed Data Links

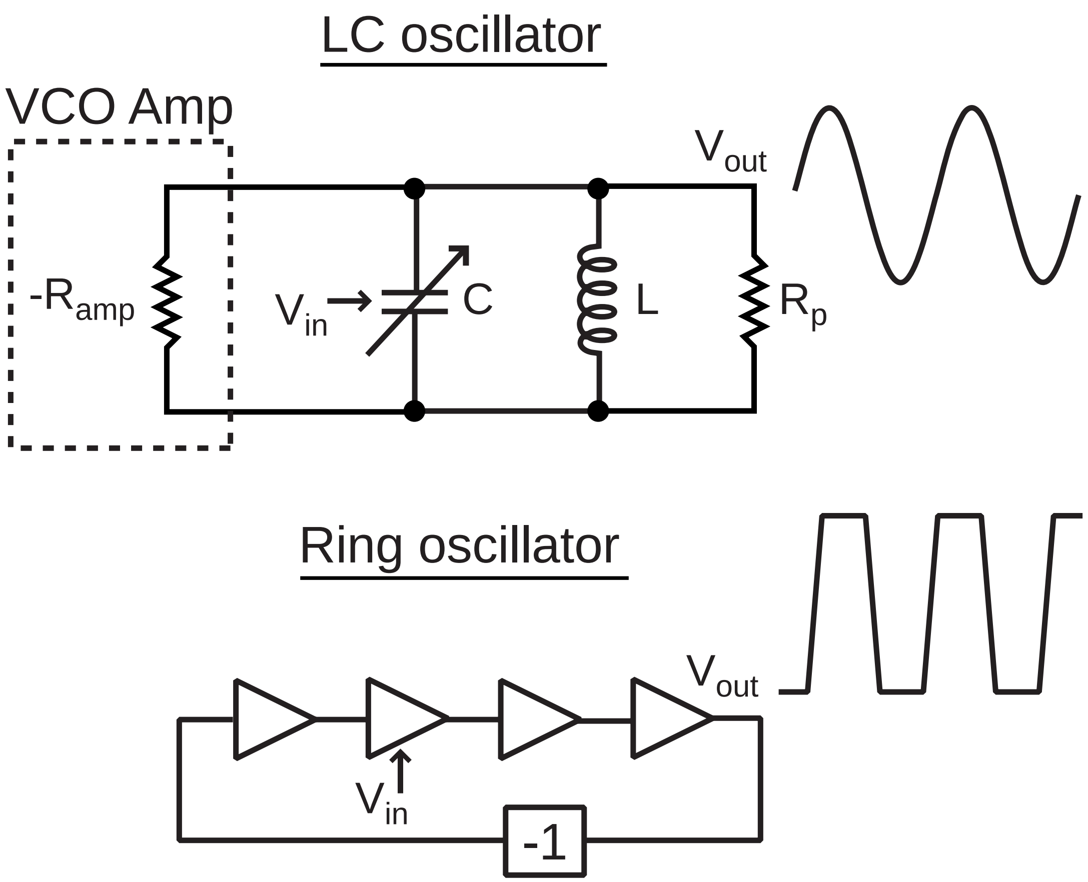


- **Design Issues**
  - **Same as wireless, but:**
    - Required noise performance is often less stringent
    - Tuning range is often narrower

Outline of Talk

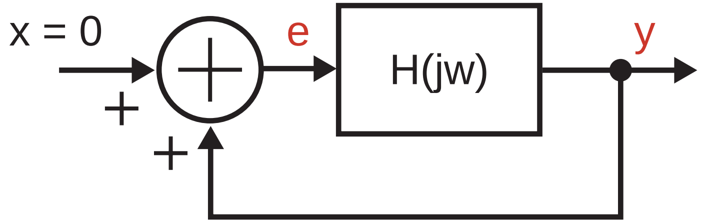
- **Common oscillator implementations**
- **Barkhausen's criterion of oscillation**
- **One-port view of resonance based oscillators**
  - **Impedance transformation**
  - **Negative feedback topologies**
- **Voltage controlled oscillators**

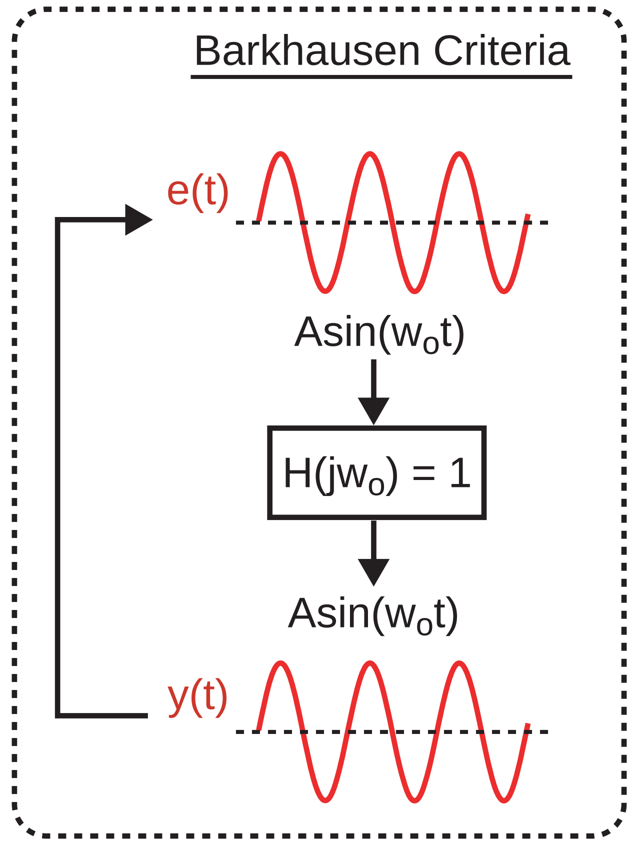
Popular VCO Structures



- **LC Oscillator: low phase noise, large area**
- **Ring Oscillator: easy to integrate, higher phase noise**

Barkhausen's Criteria for Oscillation





- **Closed loop transfer function**

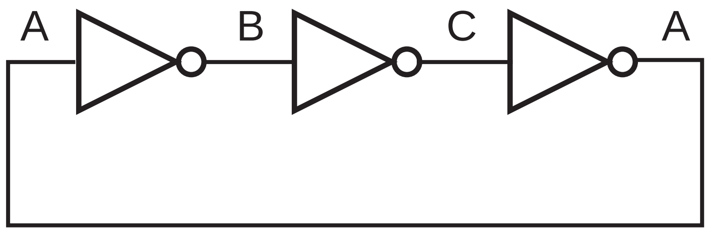
$$G(jw) = \frac{Y(jw)}{X(jw)} = \frac{H(jw)}{1 - H(jw)}$$

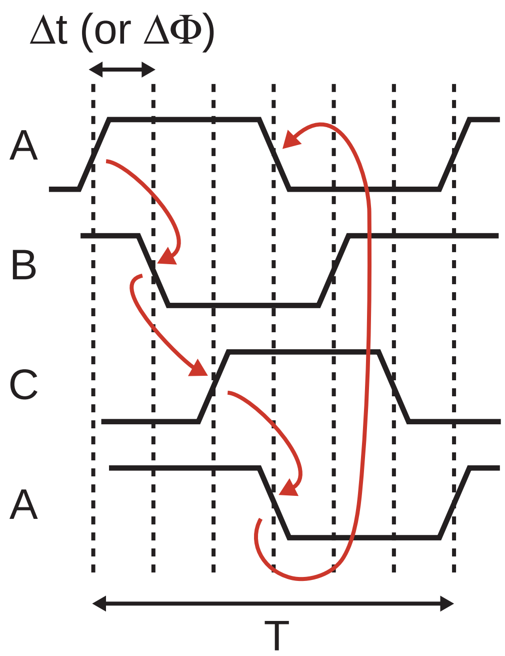
- **Self-sustaining oscillation at frequency w_o if**

$$H(jw_o) = 1$$

  - **Amounts to two conditions:**
    - Gain = 1 at frequency w_o
    - Phase = n360 degrees (n = 0,1,2,…) at frequency w_o

Example 1: Ring Oscillator





- **Gain is set to 1 by saturating characteristic of inverters**
- **Phase equals 360 degrees at frequency of oscillation**
  - **Assume N stages each with phase shift ΔΦ**

$$2N\Delta\Phi = 360^o \;\Rightarrow\; \Delta\Phi = \frac{180^o}{N}$$

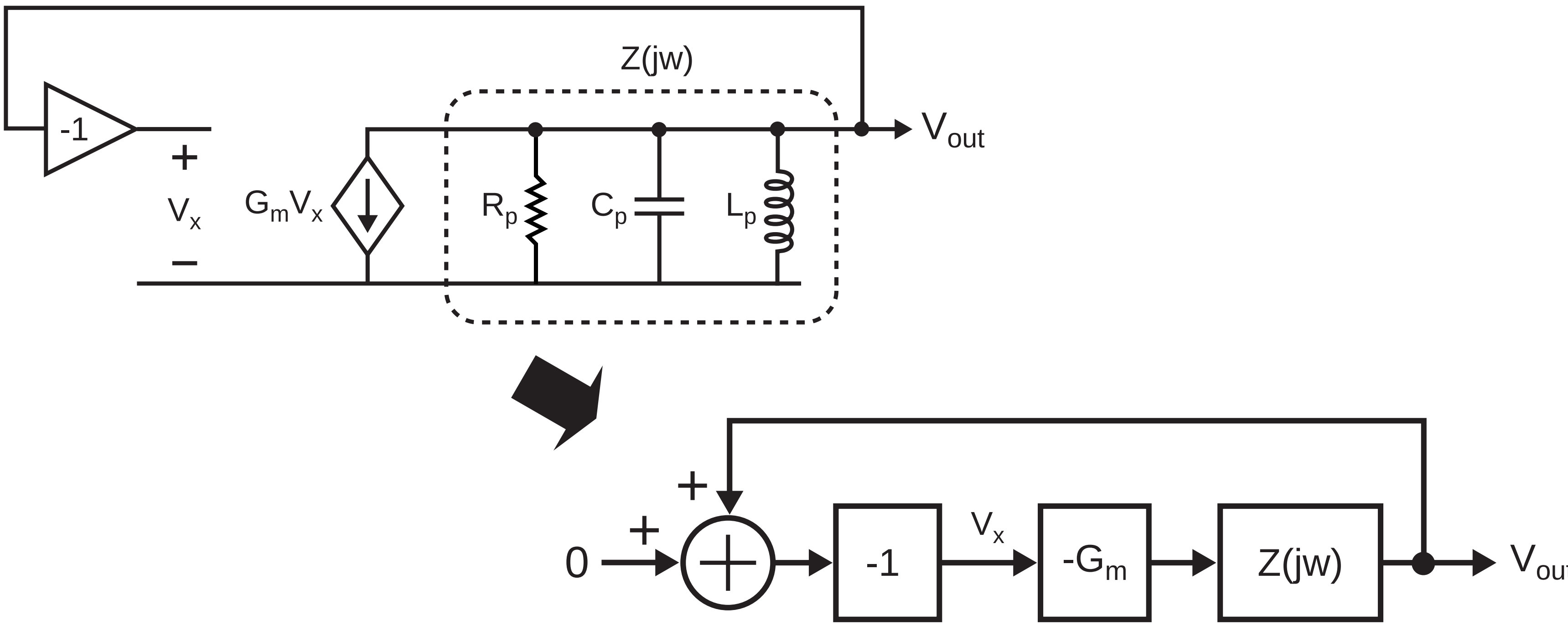
  - **Alternately, N stages with delay Δt**

$$2N\Delta t = T \;\Rightarrow\; \Delta t = \frac{T/2}{N}$$

Further Info on Ring Oscillators

- Due to their relatively poor phase noise performance, ring oscillators are rarely used in RF systems
  - They are used quite often in high speed data links, though
- We will focus on LC oscillators in this lecture
- Some useful info on CMOS ring oscillators
  - Maneatis et. al., "Precise Delay Generation Using Coupled Oscillators", JSSC, Dec 1993 (look at pp 127-128 for delay cell description)
  - Todd Weigandt's PhD thesis – http://kabuki.eecs.berkeley.edu/~weigandt/

Example 2: Resonator-Based Oscillator

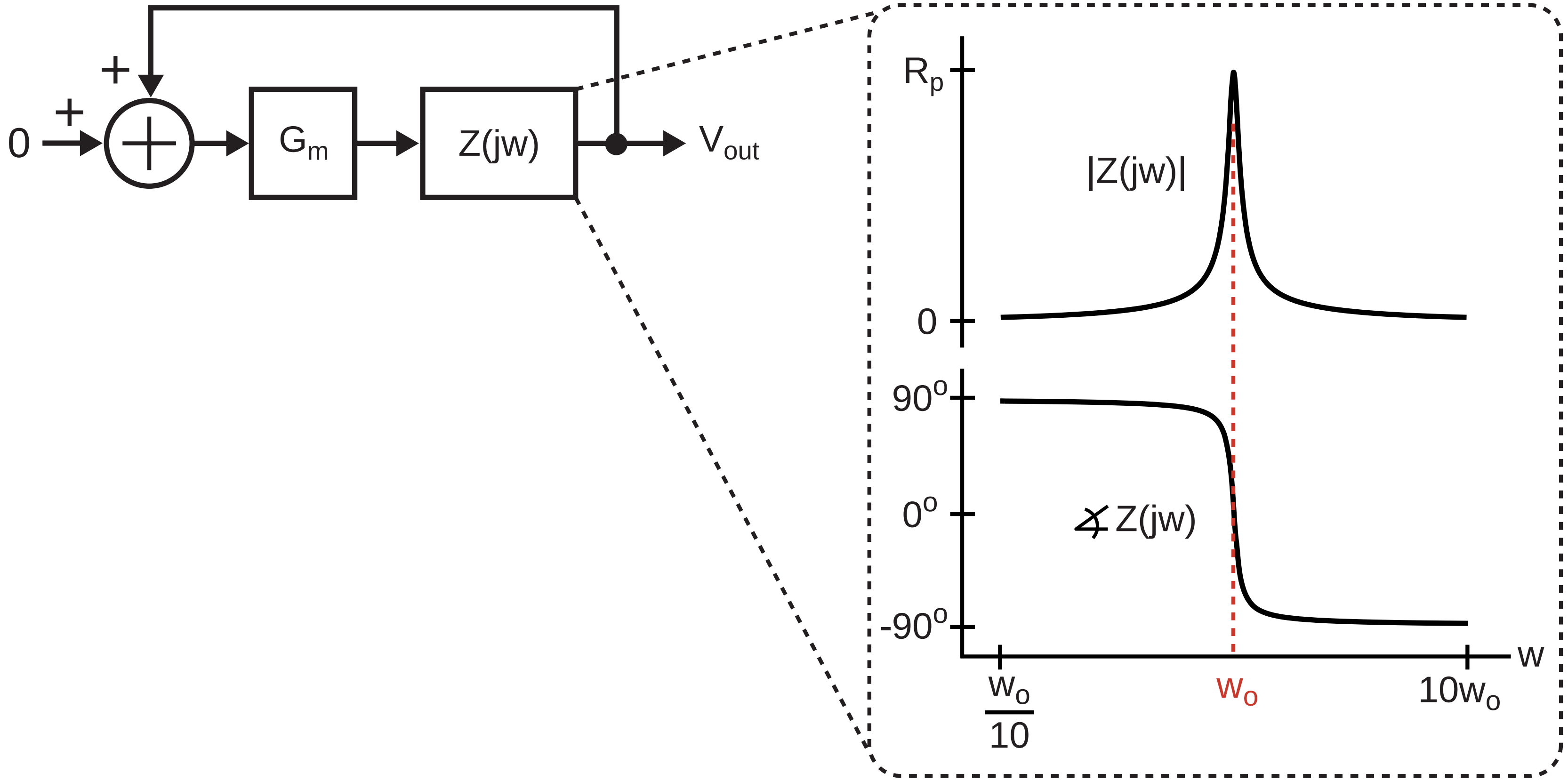


- **Barkhausen Criteria for oscillation at frequency w_o:**

$$G_m Z(jw_o) = 1$$

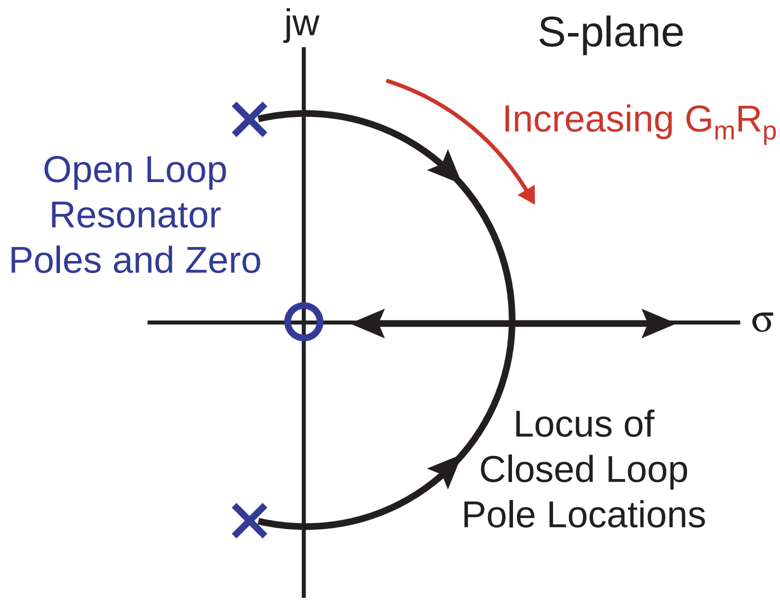
  - **Assuming G_m is purely real, $Z(jw_o)$ must also be purely real**

A Closer Look At Resonator-Based Oscillator



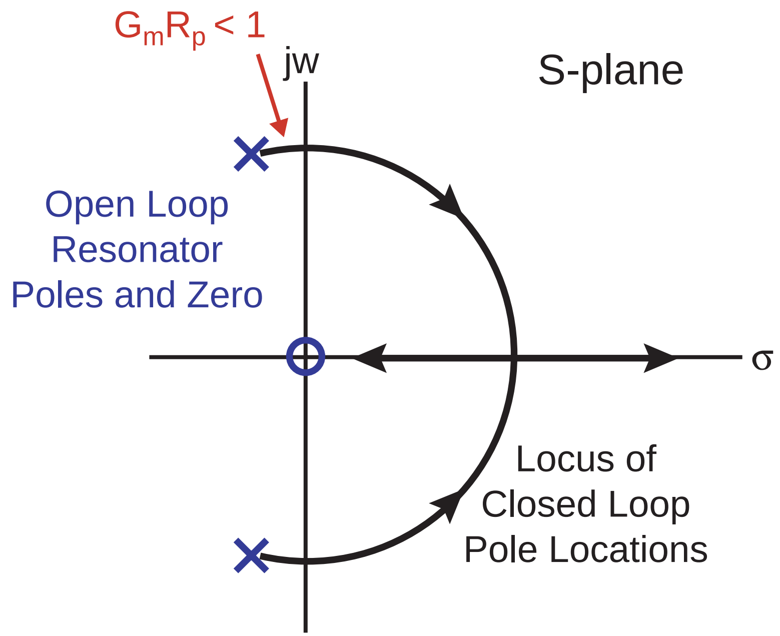
- **For parallel resonator at resonance**
  - **Looks like resistor (i.e., purely real) at resonance**
    - Phase condition is satisfied
    - Magnitude condition achieved by setting $G_mR_p = 1$

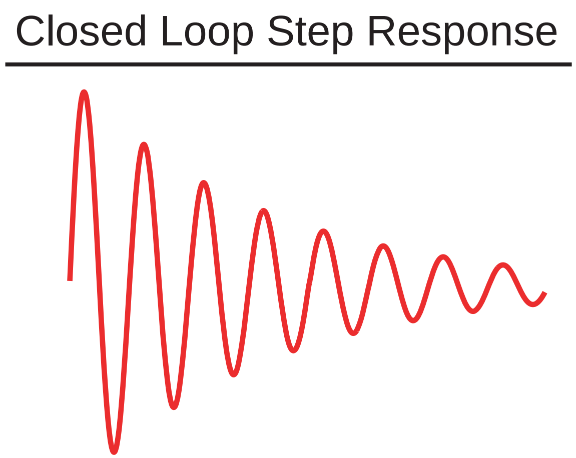
Impact of Different G_m Values



- **Root locus plot allows us to view closed loop pole locations as a function of open loop poles/zero and open loop gain (G_mR_p)**
  - **As gain (G_mR_p) increases, closed loop poles move into right half S-plane**

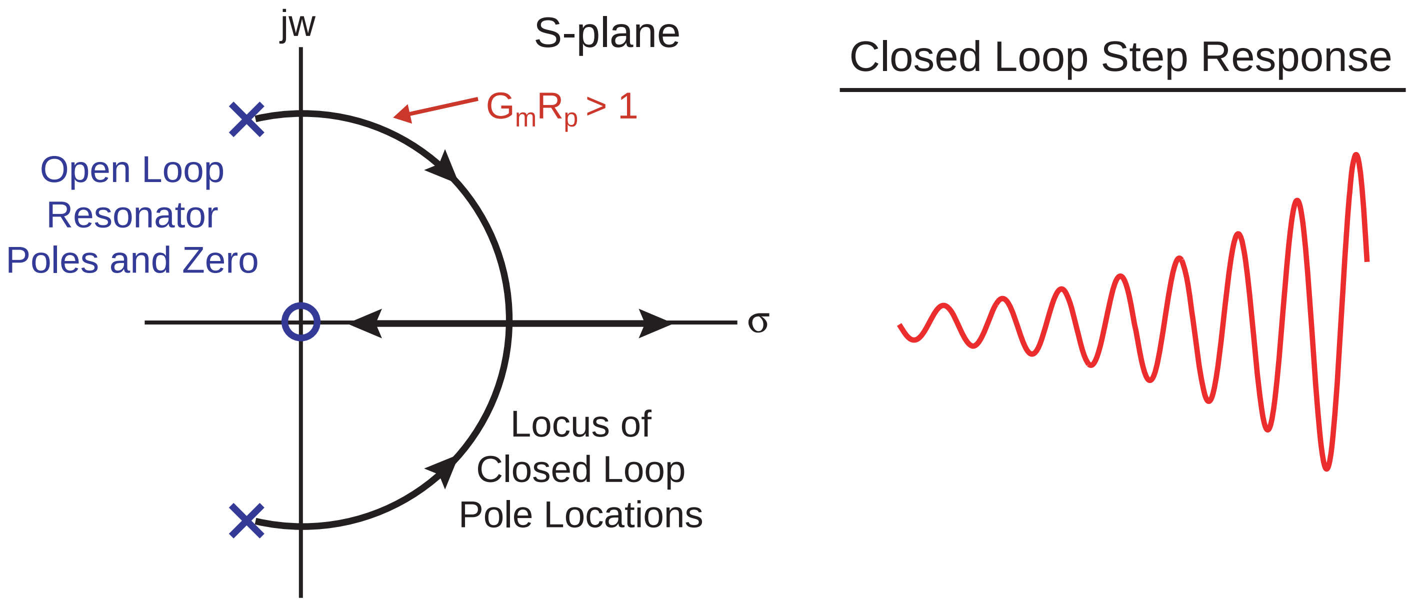
Impact of Setting G_m too low





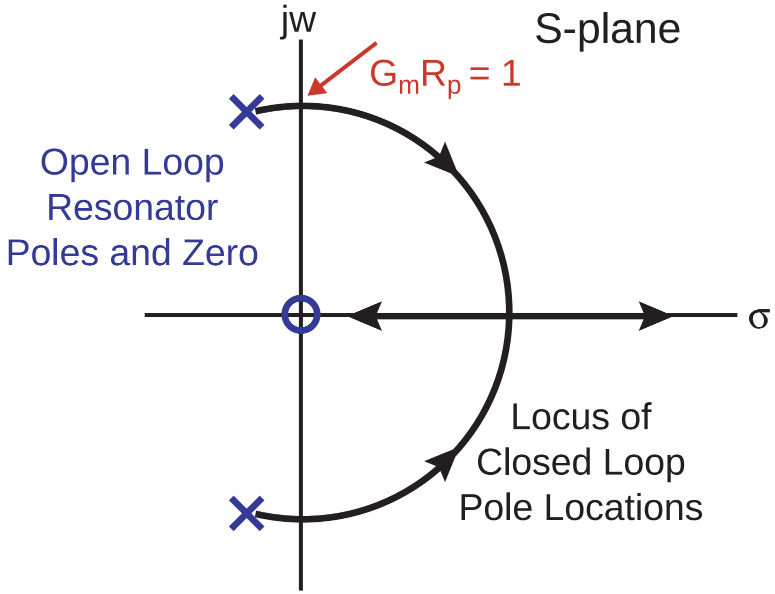
- **Closed loop poles end up in the left half S-plane**
  - **Underdamped response occurs**
    - Oscillation dies out

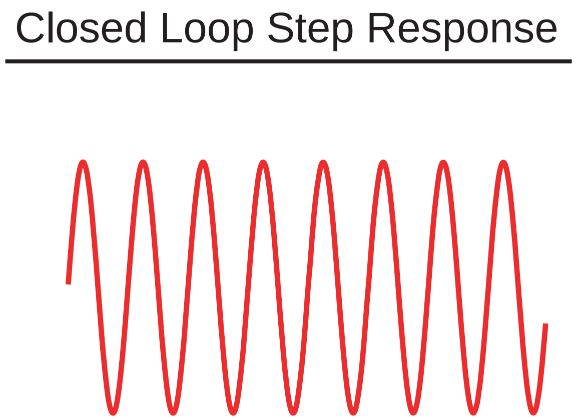
Impact of Setting G_m too High



- **Closed loop poles end up in the right half S-plane**
  - **Unstable response occurs**
    - Waveform blows up!

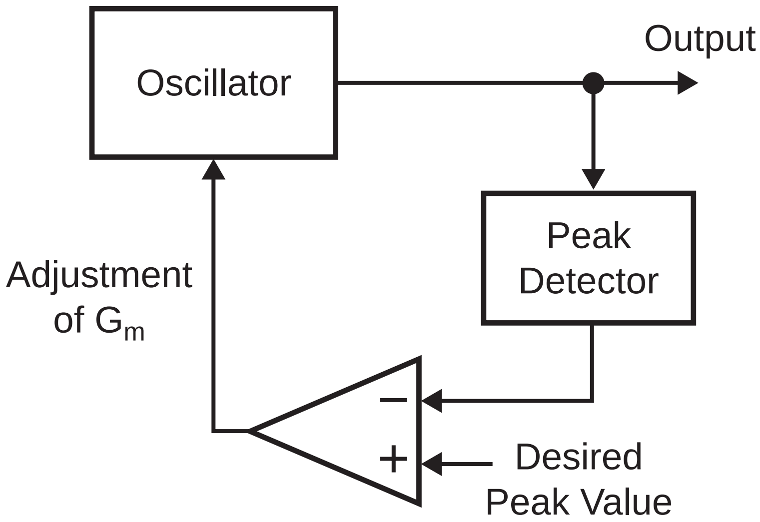
Setting G_m To Just the Right Value





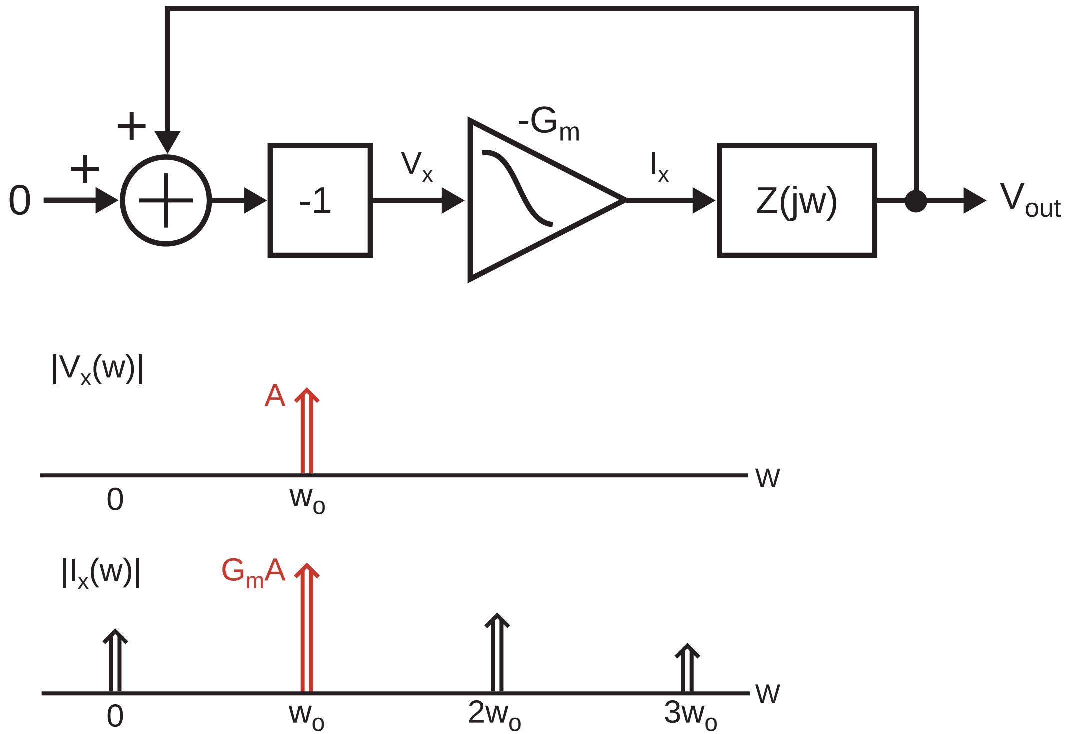
- **Closed loop poles end up on jw axis**
  - **Oscillation maintained**
- **Issue – $G_m R_p$ needs to *exactly* equal 1**
  - **How do we achieve this in practice?**

Amplitude Feedback Loop



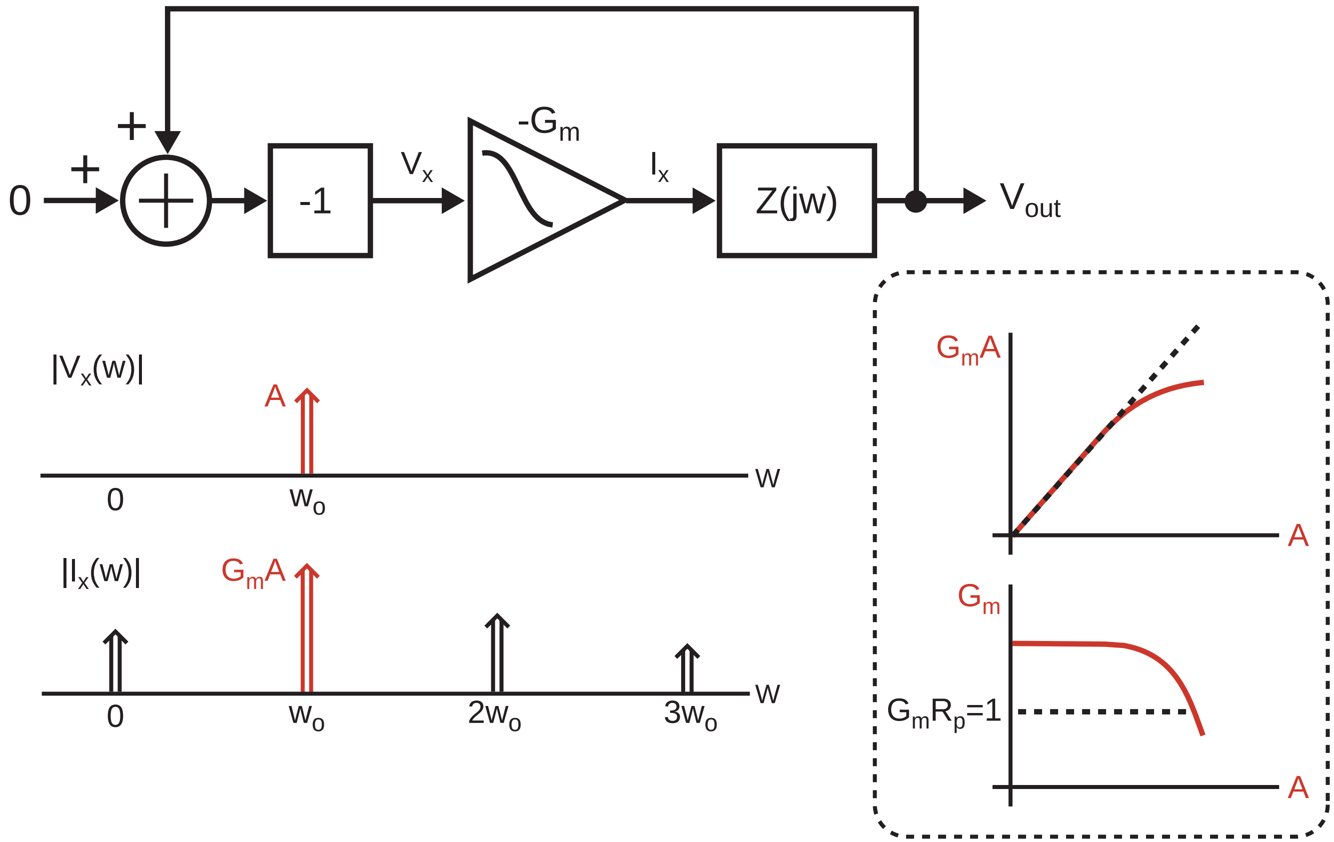
- **One thought is to detect oscillator amplitude, and then adjust G_m so that it equals a desired value**
  - **By using feedback, we can precisely achieve $G_m R_p = 1$**
- **Issues**
  - **Complex, requires power, and adds noise**

Leveraging Amplifier Nonlinearity as Feedback



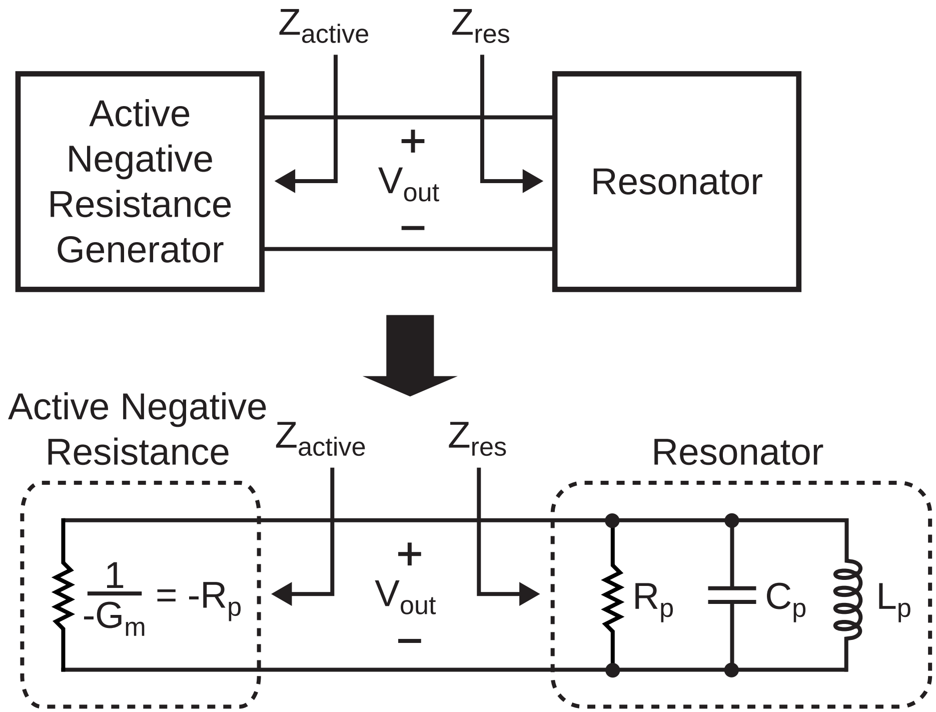
- **Practical transconductance amplifiers have saturating characteristics**
  - **Harmonics created, but filtered out by resonator**
  - **Our interest is in the relationship between the input and the fundamental of the output**

Leveraging Amplifier Nonlinearity as Feedback



- As input amplitude is increased
  - Effective gain from input to fundamental of output drops
  - Amplitude feedback occurs! (G_mR_p = 1 in steady-state)

One-Port View of Resonator-Based Oscillators

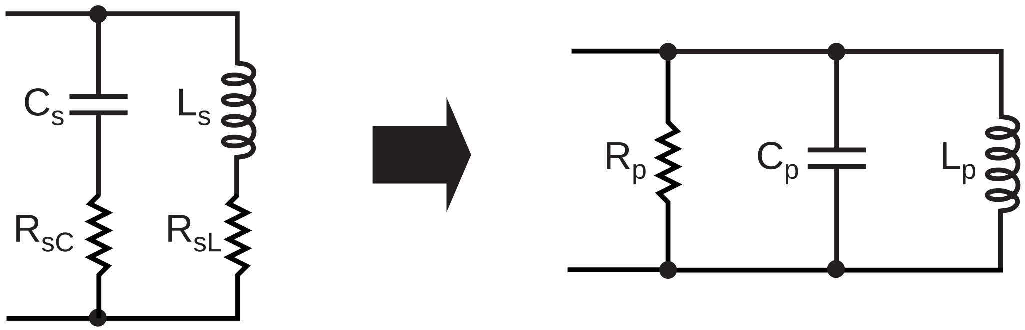


- **Convenient for intuitive analysis**
- **Here we seek to cancel out loss in tank with a negative resistance element**
  - **To achieve sustained oscillation, we must have**

$$\frac{1}{G_m} = R_p \quad \Rightarrow \quad G_m R_p = 1$$

One-Port Modeling Requires Parallel RLC Network

- **Since VCO operates over a very narrow band of frequencies, we can always do series to parallel transformations to achieve a parallel network for analysis**

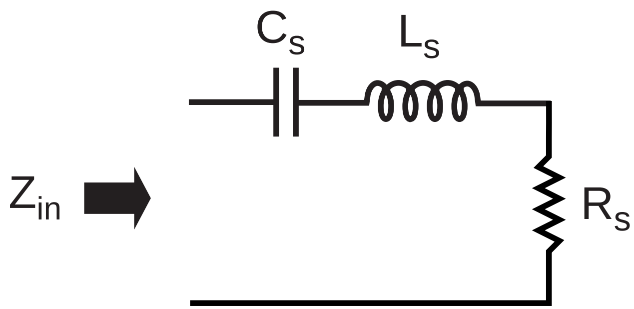


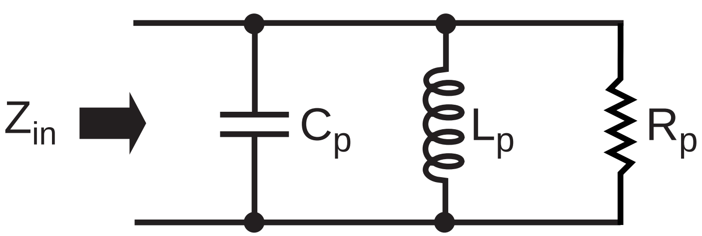
  - **Warning – in practice, RLC networks can have secondary (or more) resonant frequencies, which cause undesirable behavior**
    - Equivalent parallel network masks this problem in hand analysis
    - Simulation will reveal the problem

Understanding Narrowband Impedance Transformation

Series Resonant Circuit

Parallel Resonant Circuit





$$Z_{in} = \frac{1}{jwC_s} + jwL_s + R_s$$

$$= R_s \;\; \text{for } w = \frac{1}{\sqrt{L_sC_s}} = w_o$$

$$Q = \frac{w_oL_s}{R_s} = \frac{1}{w_oC_sR_s}$$

$$Z_{in} = \frac{1}{jwC_p}||jwL_p||R_p$$

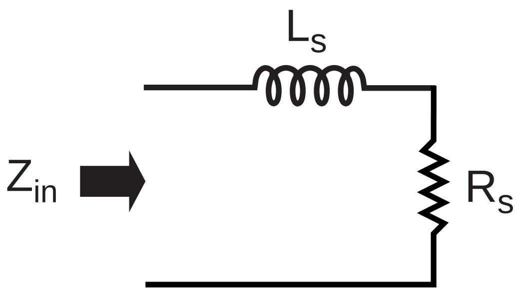
$$= R_p \;\; \text{for } w = \frac{1}{\sqrt{L_pC_p}} = w_o$$

$$Q = \frac{R_p}{w_oL_p} = w_oC_pR_p$$

- **Note: resonance allows Z_{in} to be purely real despite the presence of reactive elements**

Comparison of Series and Parallel RL Circuits

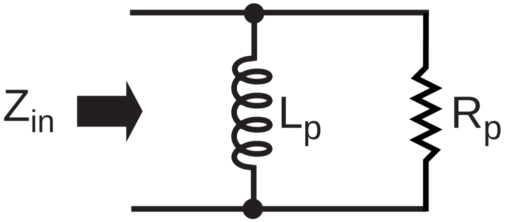
Series RL Circuit



$$Q = \frac{w_o L_s}{R_s}$$

$$Z_{in} = j w_o L_s + R_s$$

Parallel RL Circuit



$$Q = \frac{R_p}{w_o L_p}$$

$$Z_{in} = j w_o L_p || R_p$$

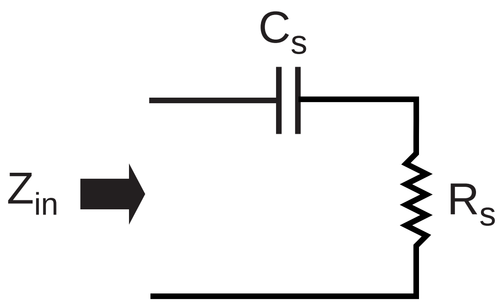
- **Equate real and imaginary parts of the left and right expressions (so that Z_{in} is the same for both)**
  - **Also equate Q values**

$$R_p = R_s(Q^2 + 1) \approx R_s Q^2 \;\; (\text{for } Q \gg 1)$$

$$L_p = L_s \left(\frac{Q^2 + 1}{Q^2}\right) \approx L_s \;\; (\text{for } Q \gg 1)$$

Comparison of Series and Parallel RC Circuits

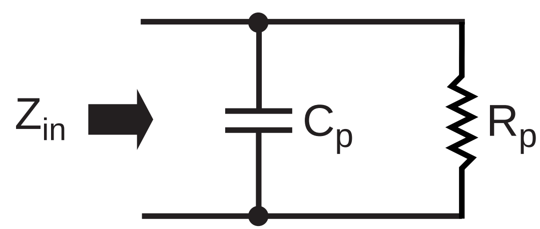
Series RC Circuit



$$Q = \frac{1}{w_o C_s R_s}$$

$$Z_{in} = R_s + \frac{1}{j w_o C_s}$$

Parallel RC Circuit



$$Q = w_o C_p R_p$$

$$Z_{in} = R_p || \frac{1}{j w_o C_p}$$

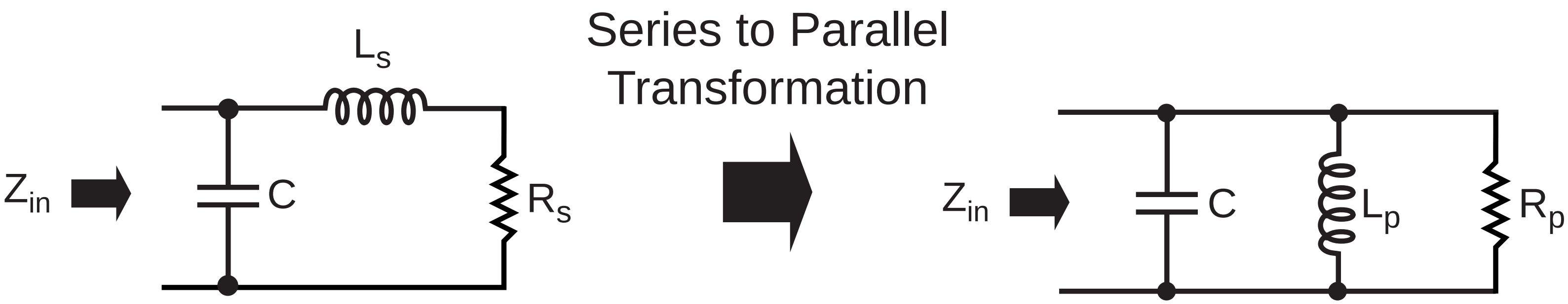
- **Equate real and imaginary parts of the left and right expressions (so that Z_{in} is the same for both)**
  - **Also equate Q values**

$$R_p = R_s(Q^2 + 1) \approx R_s Q^2 \;\; (\text{for } Q \gg 1)$$

$$C_p = C_s \left(\frac{Q^2}{Q^2 + 1} \right) \approx C_s \;\; (\text{for } Q \gg 1)$$

Example Transformation to Parallel RLC

- **Assume Q >> 1**



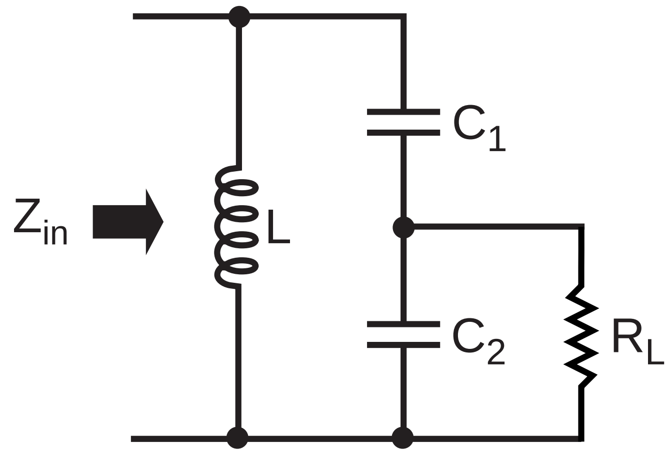
$$L_p \approx L_s$$

$$R_p \approx Q^2 R_s$$

- **Note at resonance:**

$$Z_{in} = R_p \approx Q^2 R_s \quad \text{(purely real)}$$

Tapped Capacitor as a Transformer

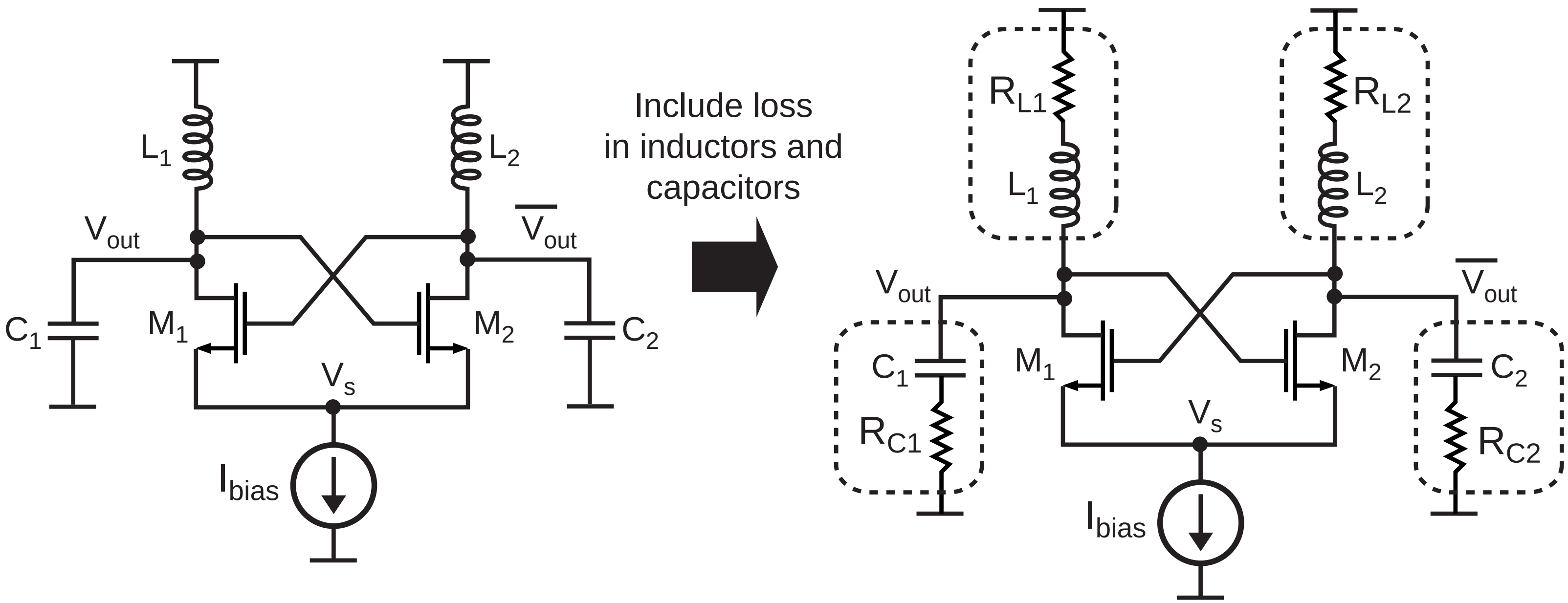


- **To first order:**

$$\frac{R_{in}}{R_L} \approx \left(\frac{C_1 + C_2}{C_1}\right)^2$$

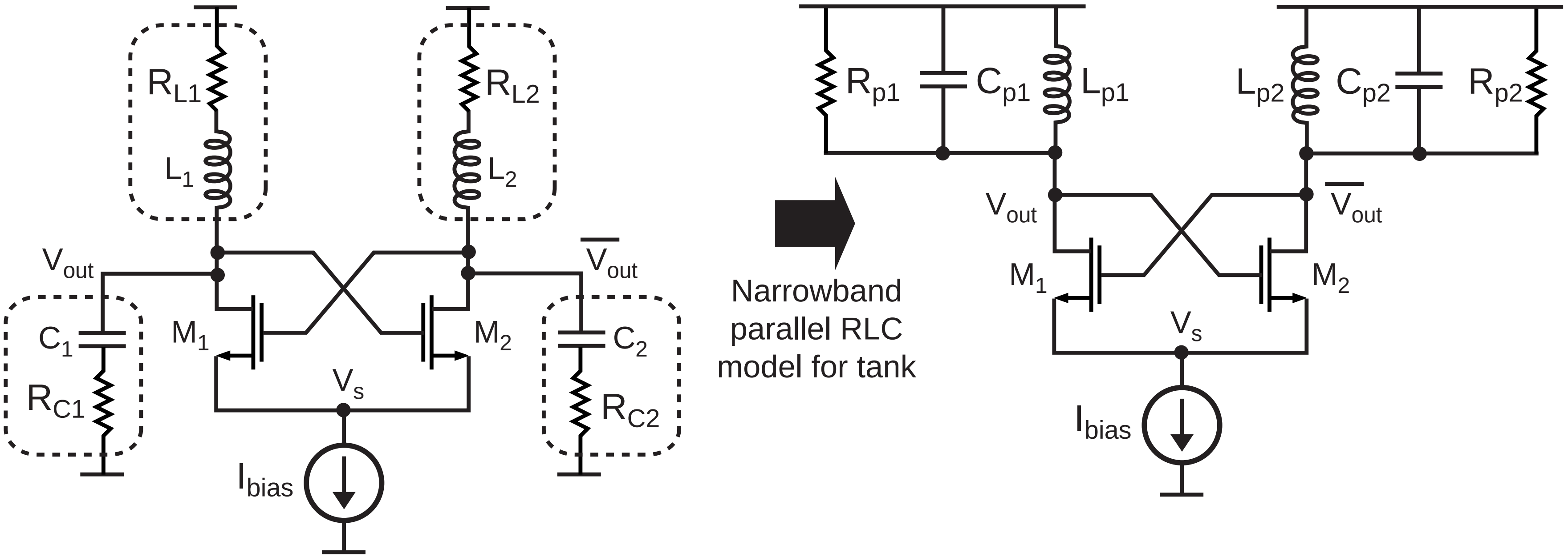
- **We will see this used in Colpitts oscillator**

Negative Resistance Oscillator



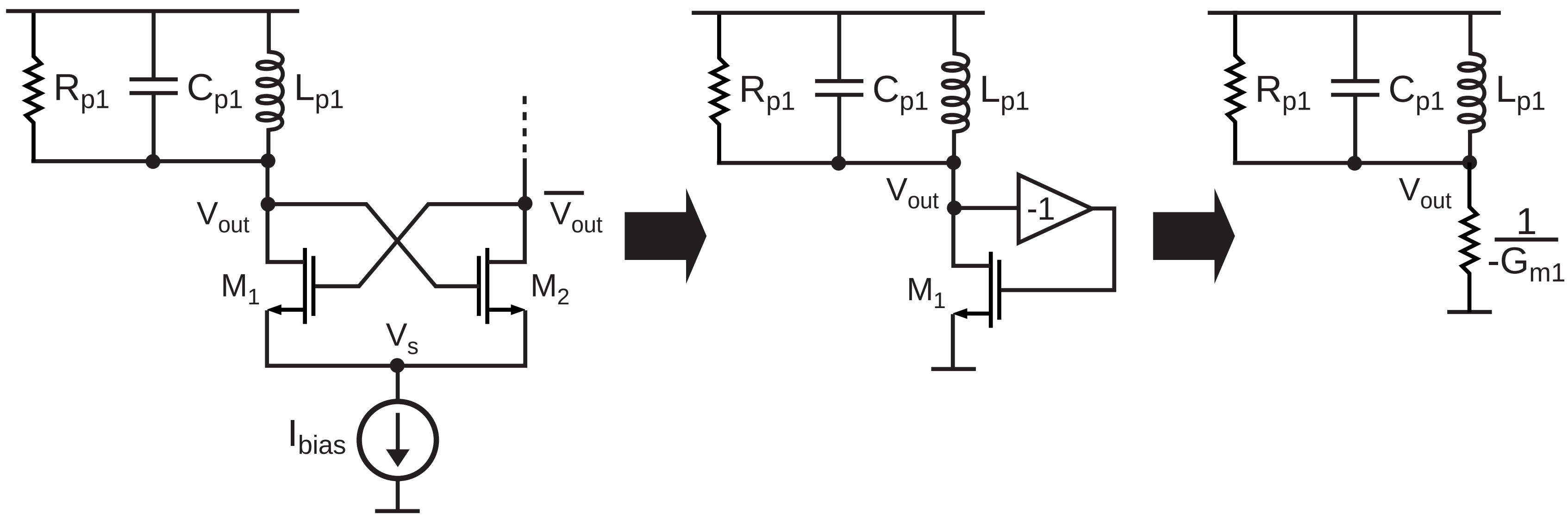
- **This type of oscillator structure is quite popular in current CMOS implementations**
  - **Advantages**
    - Simple topology
    - Differential implementation (good for feeding differential circuits)
    - Good phase noise performance can be achieved

Analysis of Negative Resistance Oscillator (Step 1)



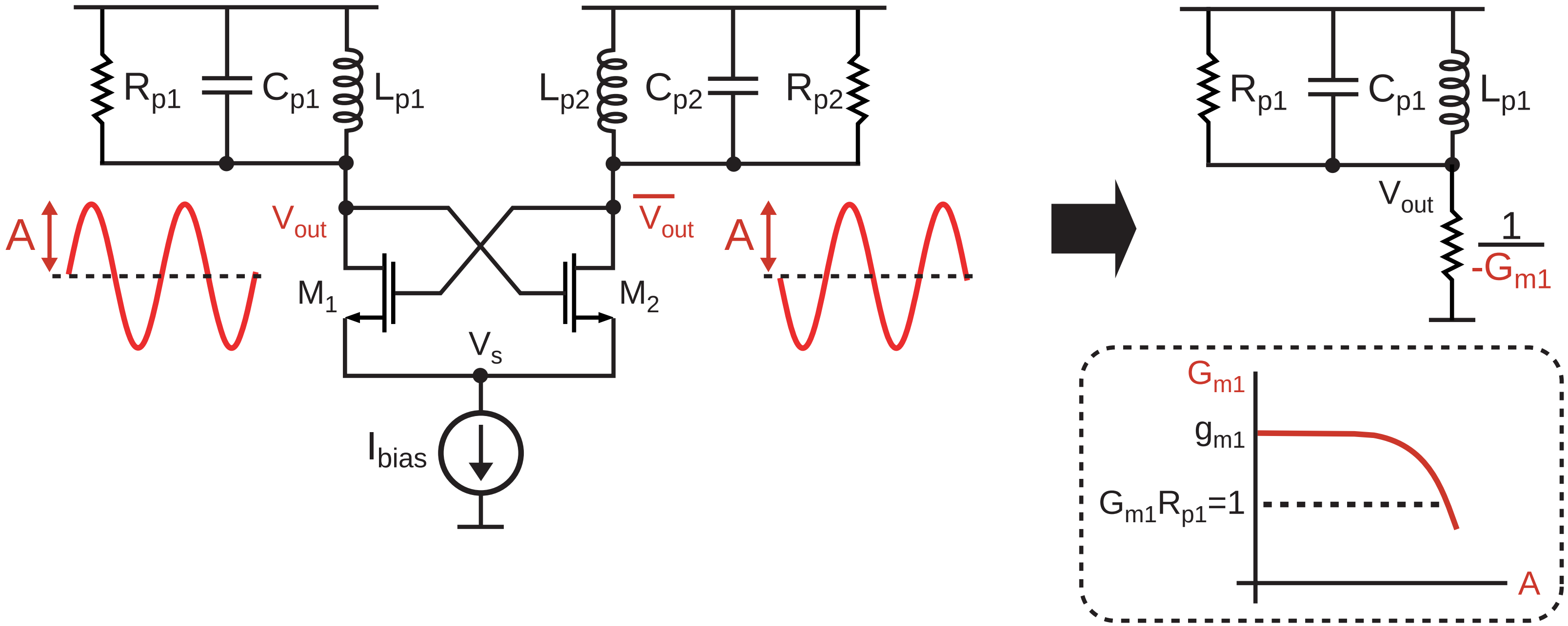
- **Derive a parallel RLC network that includes the loss of the tank inductor and capacitor**
  - **Typically, such loss is dominated by series resistance in the inductor**

Analysis of Negative Resistance Oscillator (Step 2)



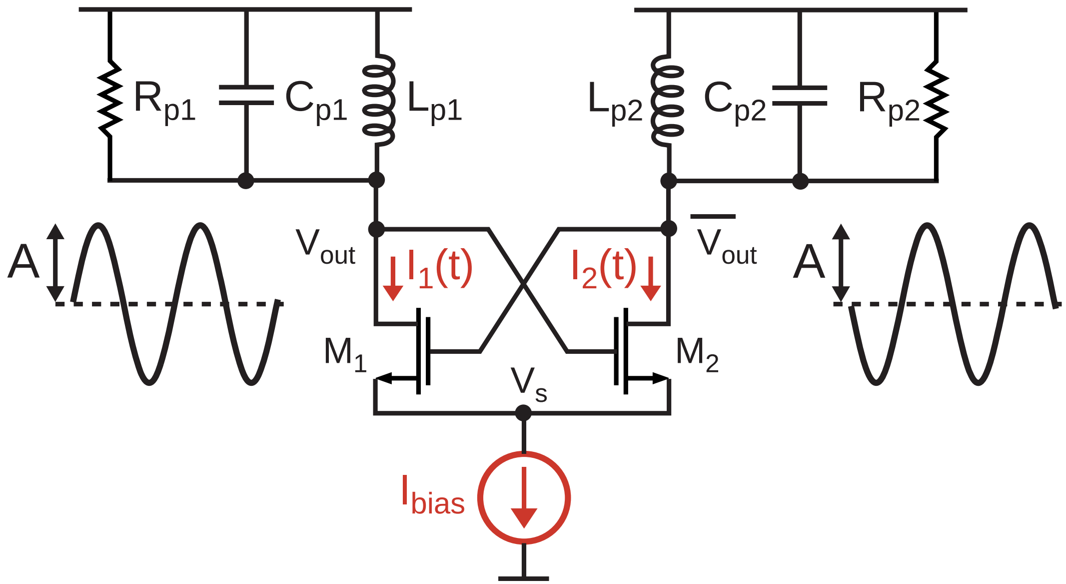
- **Split oscillator circuit into half circuits to simplify analysis**
  - **Leverages the fact that we can approximate V_s as being incremental ground (this is not quite true, but close enough)**
- **Recognize that we have a diode connected device with a negative transconductance value**
  - **Replace with negative resistor**
    - Note: G_m is ***large signal*** transconductance value

Design of Negative Resistance Oscillator



- **Design tank components to achieve high Q**
  - **Resulting R_p value is as large as possible**
- **Choose bias current (I_{bias}) for large swing (without going far into saturation)**
  - **We'll estimate swing as a function of I_{bias} shortly**
- **Choose transistor size to achieve adequately large g_{m1}**
  - **Usually twice as large as $1/R_{p1}$ to guarantee startup**

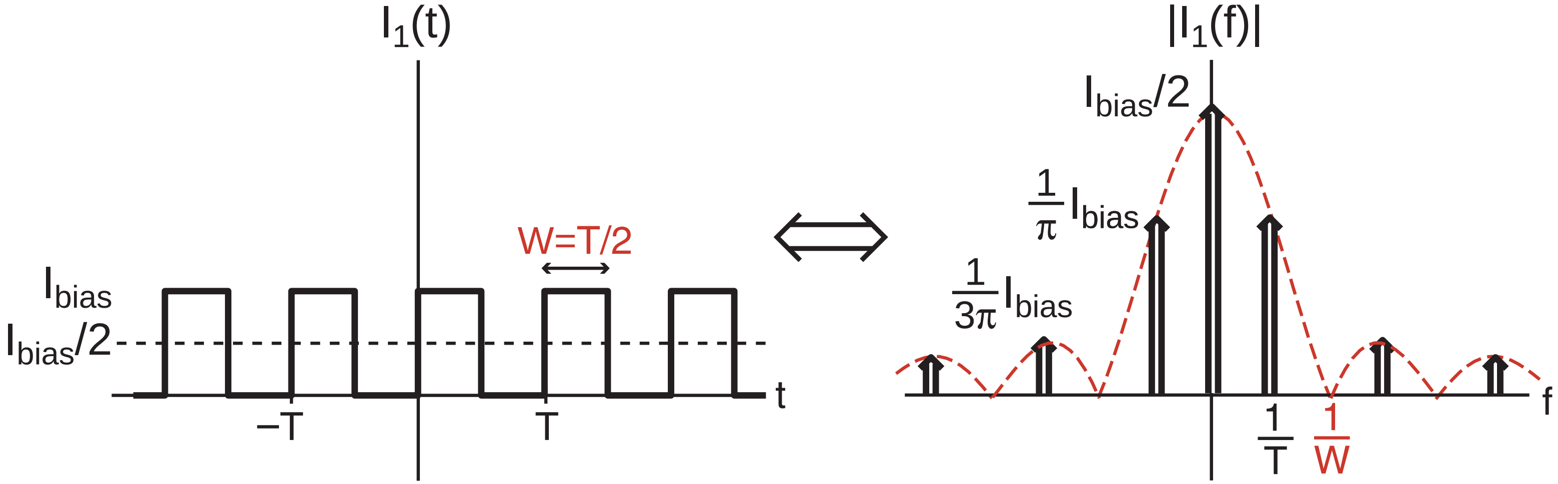
Calculation of Oscillator Swing



- Design tank components to achieve high Q
  - Resulting R_p value is as large as possible
- Choose bias current (I_{bias}) for large swing (without going far into saturation)
  - We'll estimate swing as a function of I_{bias} in next slide
- Choose transistor size to achieve adequately large g_{m1}
  - Usually twice as large as $1/R_{p1}$ to guarantee startup

Calculation of Oscillator Swing as a Function of I_{bias}

- **By symmetry, assume $I_1(t)$ is a square wave**
  - **We are interested in determining fundamental component**
    - (DC and harmonics filtered by tank)



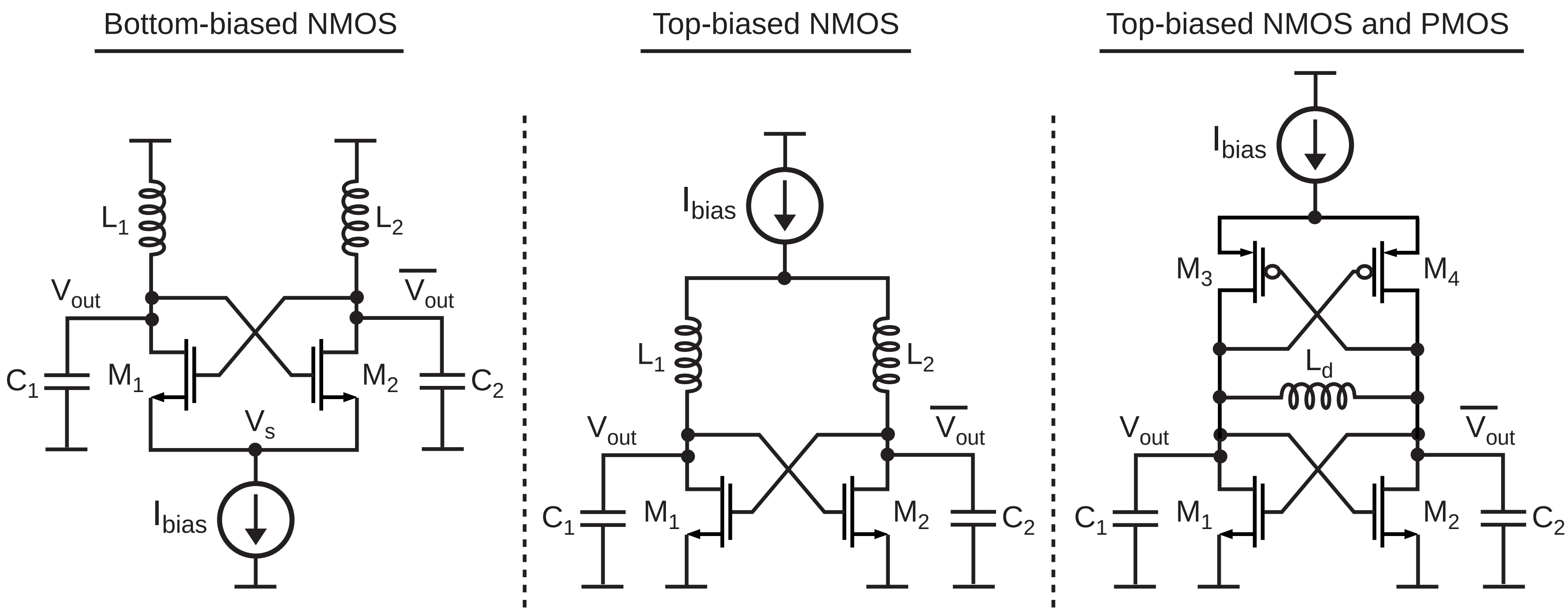
  - **Fundamental component is**

$$I_1(t)\Big|_{fundamental} = \frac{2}{\pi} I_{bias} \sin(w_o t), \quad \text{where} \quad w_o = \frac{2\pi}{T}$$

  - **Resulting oscillator amplitude**

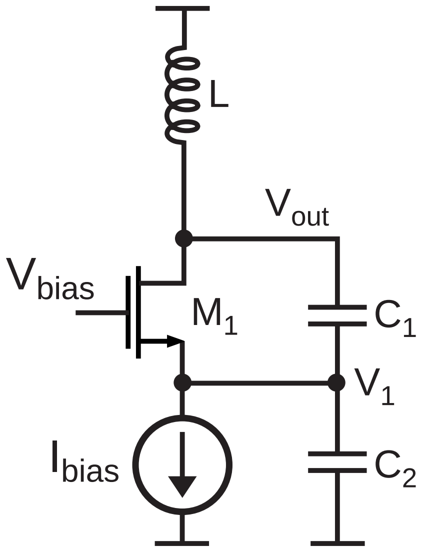
$$A = \frac{2}{\pi} I_{bias} R_p$$

Variations on a Theme



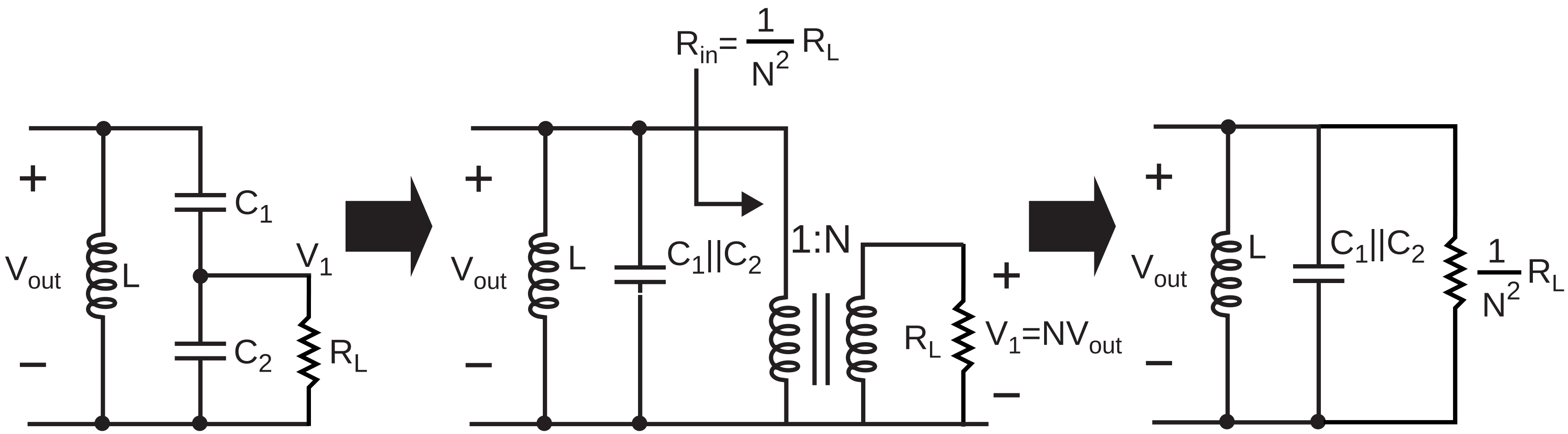
- **Biasing can come from top or bottom**
- **Can use either NMOS, PMOS, or both for transconductor**
  - **Use of both NMOS and PMOS for coupled pair would appear to achieve better phase noise at a given power dissipation**
    - See Hajimiri et. al, “Design Issues in CMOS Differential LC Oscillators”, JSSC, May 1999 and Feb, 2000 (pp 286-287)

Colpitts Oscillator



- **Carryover from discrete designs in which single-ended approaches were preferred for simplicity**
  - **Achieves negative resistance with only one transistor**
  - **Differential structure can also be implemented**
- **Good phase noise can be achieved, but not apparent there is an advantage of this design over negative resistance design for CMOS applications**

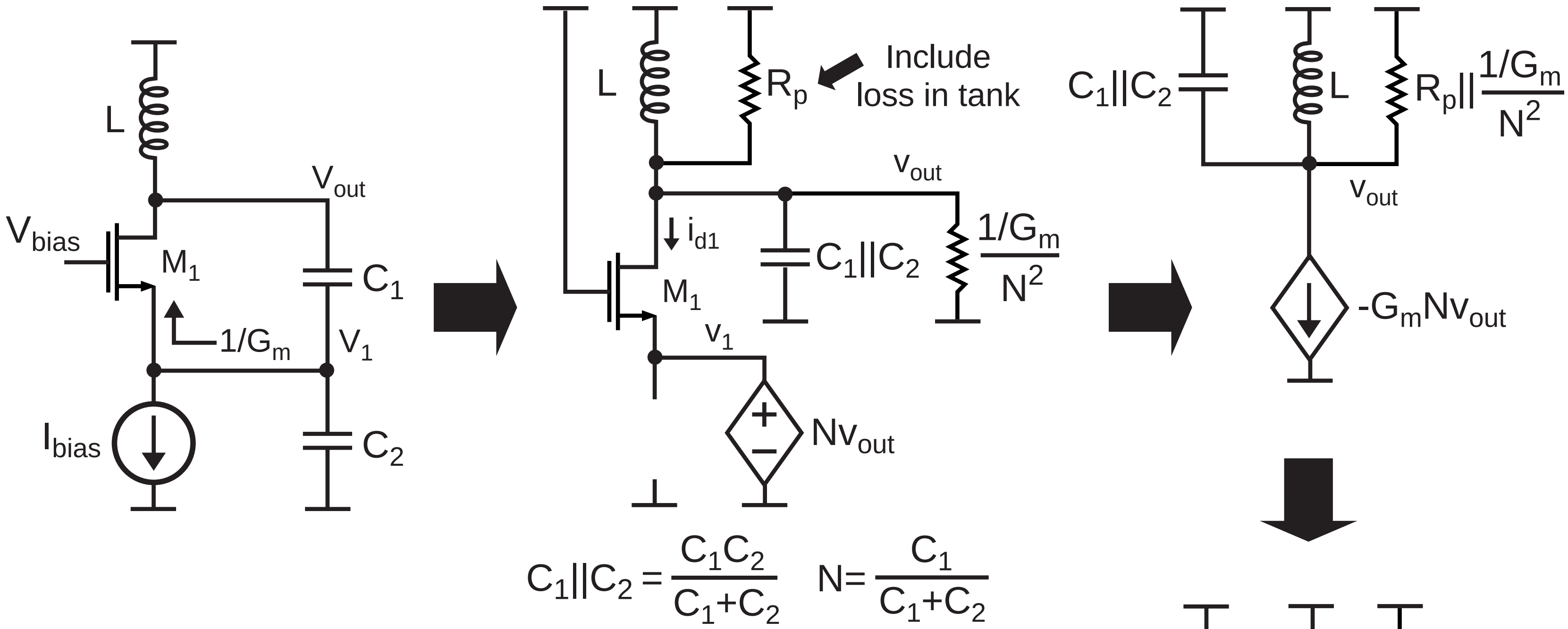
Analysis of Cap Transformer used in Colpitts

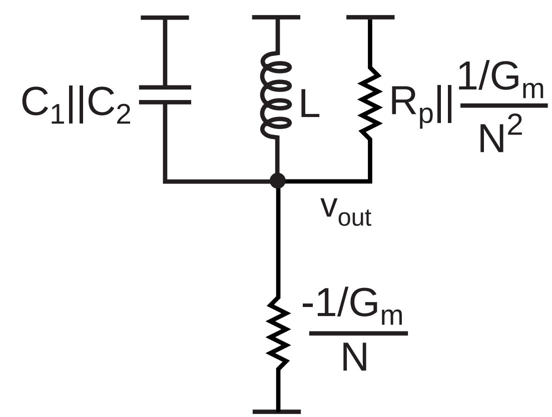


$$C_1||C_2 = \frac{C_1C_2}{C_1+C_2} \qquad N = \frac{C_1}{C_1+C_2}$$

- **Voltage drop across R_L is reduced by capacitive voltage divider**
  - **Assume that impedances of caps are less than R_L at resonant frequency of tank (simplifies analysis)**
    - Ratio of V_1 to V_{out} set by caps and not R_L
- **Power conservation leads to transformer relationship shown**

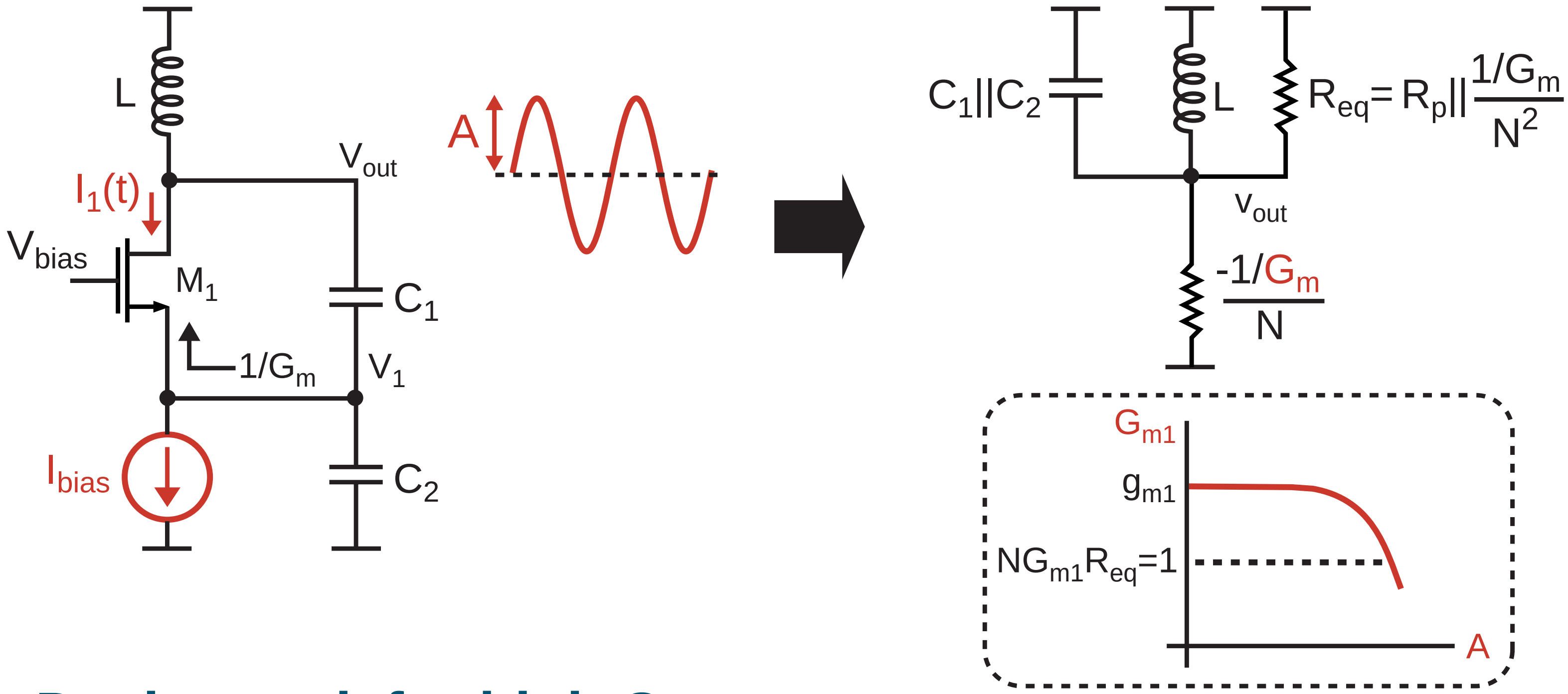
Simplified Model of Colpitts





- **Purpose of cap transformer**
  - **Reduces loading on tank**
  - **Reduces swing at source node (important for bipolar version)**
- **Transformer ratio set to achieve best noise performance**

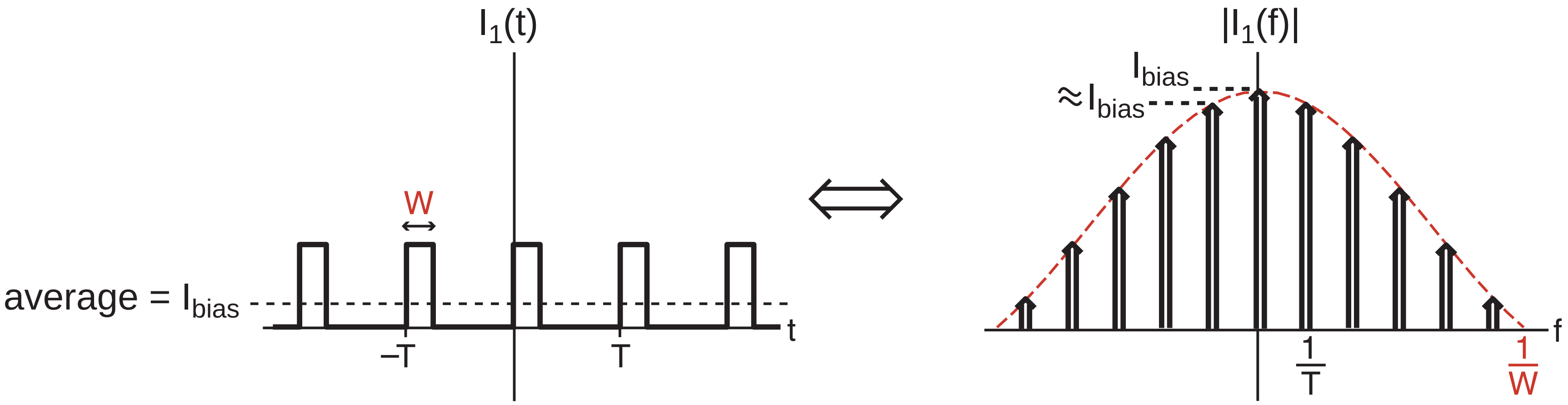
Design of Colpitts Oscillator



- **Design tank for high Q**
- **Choose bias current (I_{bias}) for large swing (without going far into saturation)**
- **Choose transformer ratio for best noise**
  - **Rule of thumb: choose N = 1/5 according to Tom Lee**
- **Choose transistor size to achieve adequately large g_{m1}**

Calculation of Oscillator Swing as a Function of I_{bias}

- $I_1(t)$ consists of pulses whose shape and width are a function of the transistor behavior and transformer ratio
  - Approximate as narrow square wave pulses with width W



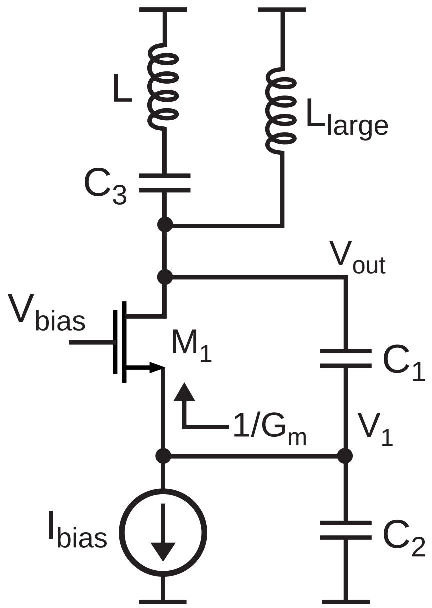
  - Fundamental component is

$$I_1(t)\Big|_{fundamental} \approx 2I_{bias}\sin(w_o t), \quad \text{where} \;\; w_o = \frac{2\pi}{T}$$

  - Resulting oscillator amplitude

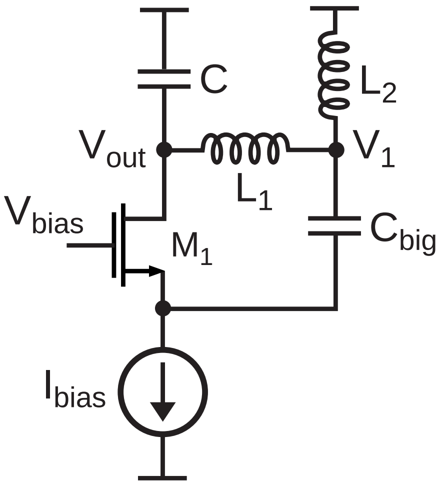
$$A \approx 2I_{bias}R_{eq}$$

Clapp Oscillator



- **Same as Colpitts except that inductor portion of tank is isolated from the drain of the device**
  - **Allows inductor voltage to achieve a larger amplitude without exceeded the max allowable voltage at the drain**
    - Good for achieving lower phase noise

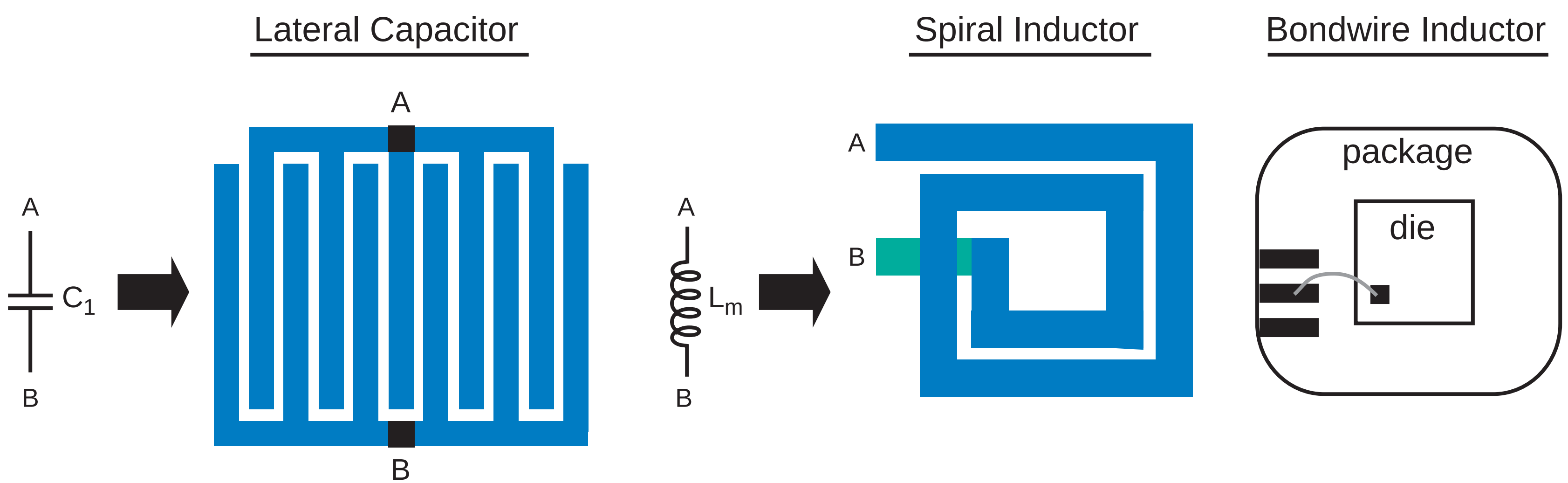
Hartley Oscillator



- **Same as Colpitts, but uses a tapped inductor rather than series capacitors to implement the transformer portion of the circuit**
  - **Not popular for IC implementations due to the fact that capacitors are easier to realize than inductors**

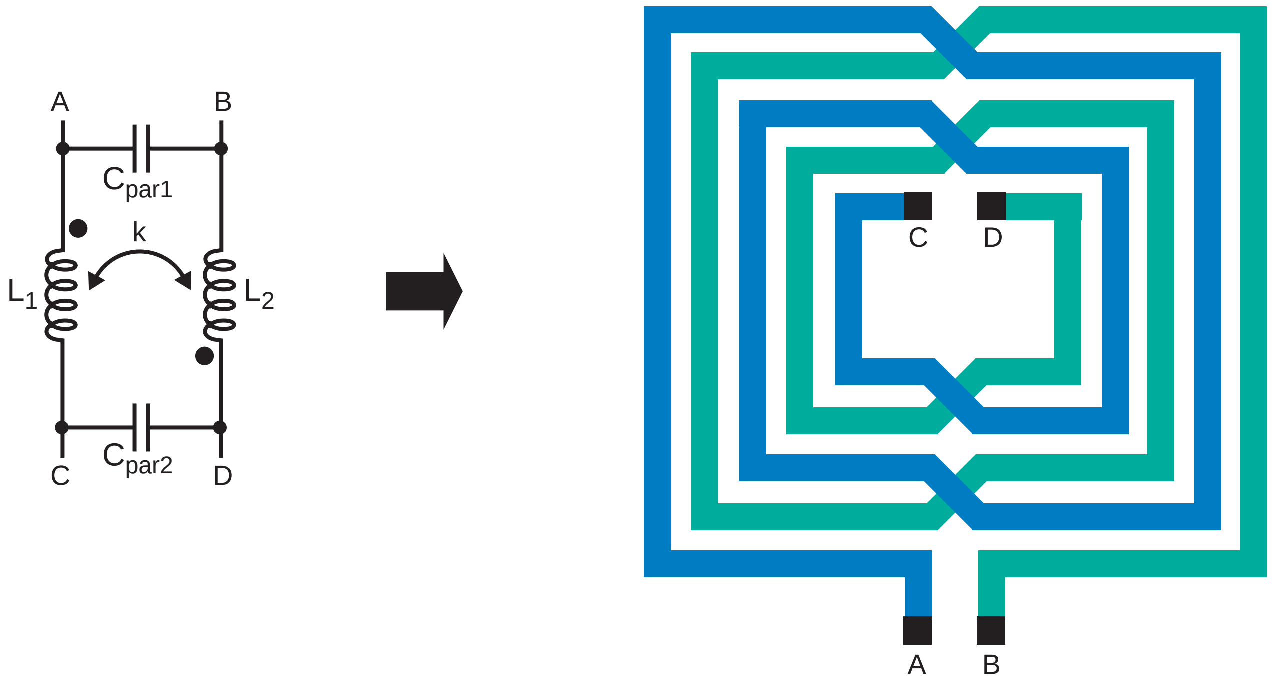
Integrated Resonator Structures

- **Inductor and capacitor tank**
  - **Lateral caps have high Q (> 50)**
  - **Spiral inductors have moderate Q (5 to 10), but completely integrated and have tight tolerance (< § 10%)**
  - **Bondwire inductors have high Q (> 40), but not as “integrated” and have poor tolerance (> § 20%)**

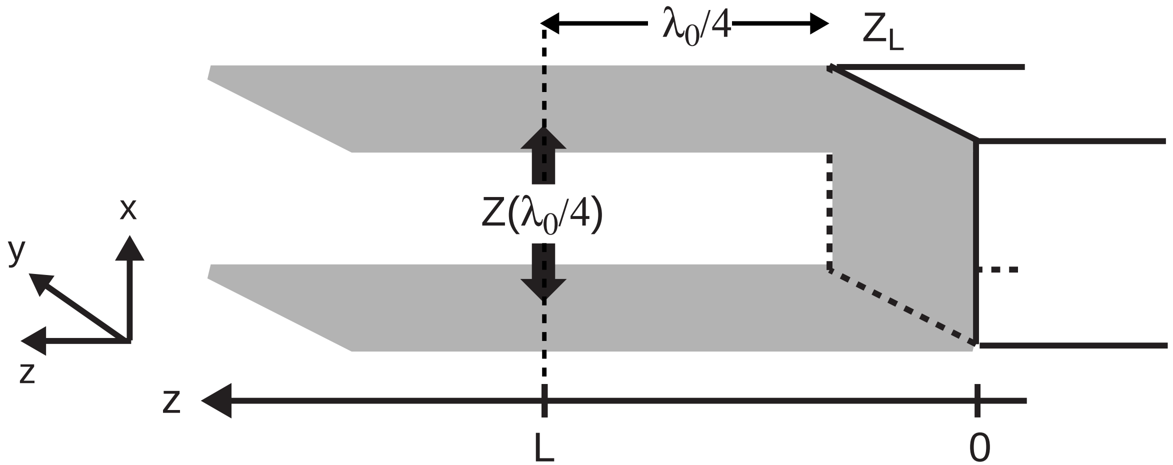


Integrated Resonator Structures

- **Integrated transformer**
  - **Leverages self and mutual inductance for resonance to achieve higher Q**
  - **See Straayer et. al., “A low-noise transformer-based 1.7 GHz CMOS VCO”, ISSCC 2002, pp 286-287**



Quarter Wave Resonator



- **Impedance calculation (from Lecture 4)**

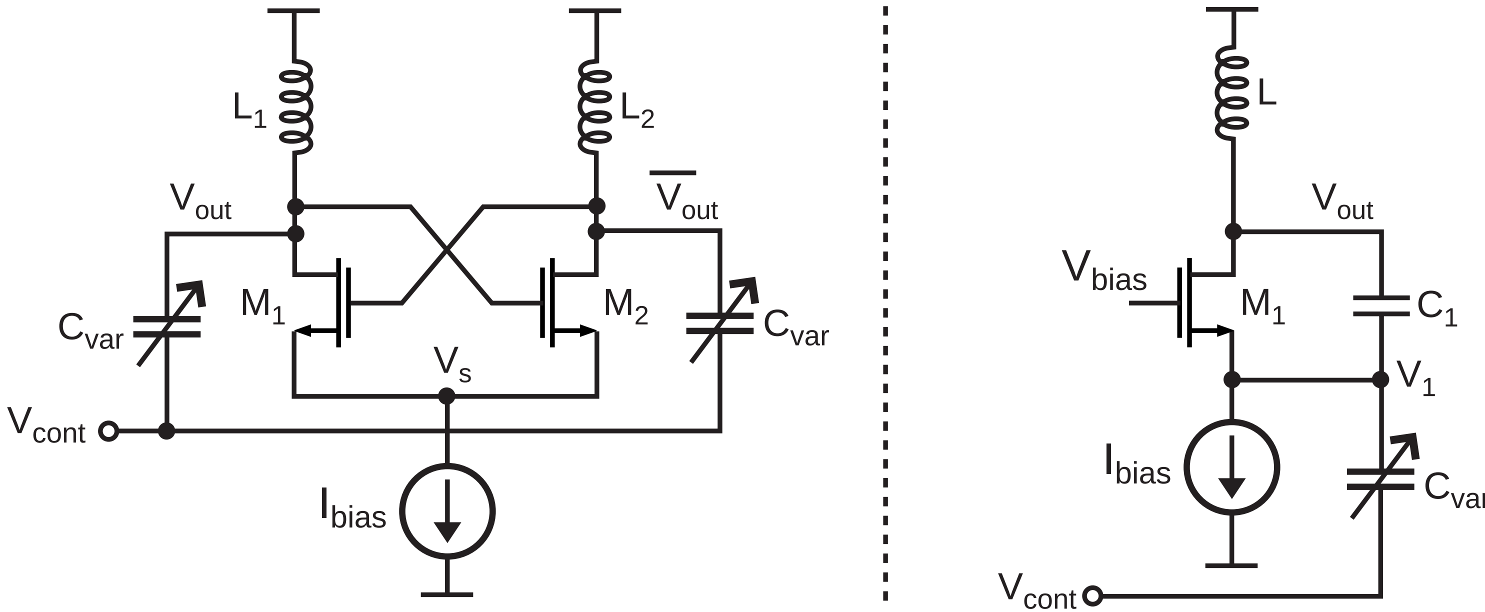
$$Z(\lambda_o/4) \approx -j\frac{2}{\pi}\sqrt{\frac{L}{C}}\left(\frac{w_o}{\Delta w}\right)$$

  - **Looks like parallel LC tank!**
- **Benefit – very high Q can be achieved with fancy dielectric**
- **Negative – relatively large area (external implementation in the past), but getting smaller with higher frequencies!**

Other Types of Resonators

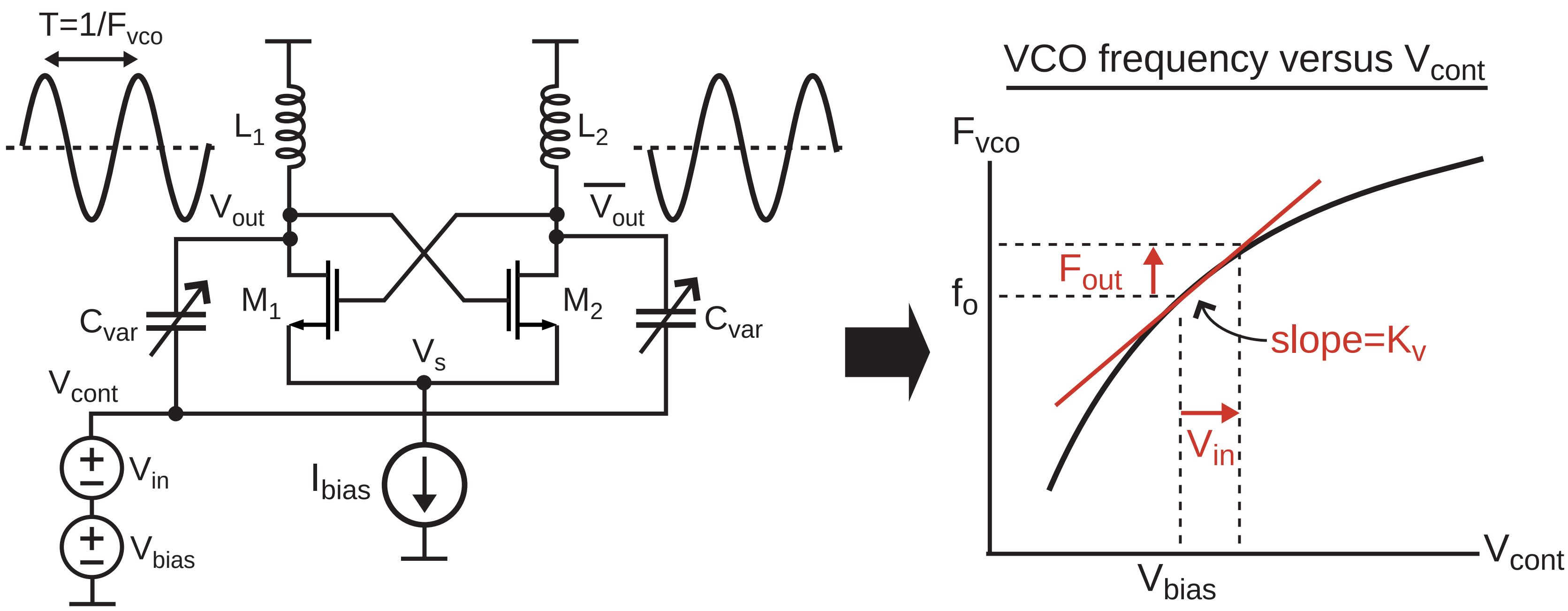
- **Quartz crystal**
  - **Very high Q, and very accurate and stable resonant frequency**
    - Confined to low frequencies (< 200 MHz)
    - Non-integrated
  - **Used to create low noise, accurate, “reference” oscillators**
- **SAW devices**
  - **High frequency, but poor accuracy (for resonant frequency)**
- **MEMS devices**
  - **Cantilever beams – promise high Q, but non-tunable and haven’t made it to the GHz range, yet, for resonant frequency**
  - **FBAR – Q > 1000, but non-tunable and poor accuracy**
  - **More on this topic in the last lecture this week**

Voltage Controlled Oscillators (VCO's)



- **Include a tuning element to adjust oscillation frequency**
  - **Typically use a variable capacitor (varactor)**
- **Varactor incorporated by replacing fixed capacitance**
  - **Note that much fixed capacitance cannot be removed (transistor junctions, interconnect, etc.)**
    - Fixed cap lowers frequency tuning range

Model for Voltage to Frequency Mapping of VCO



- Model VCO in a small signal manner by looking at deviations in frequency about the bias point
  - Assume linear relationship between input voltage and output frequency

$$F_{out}(t) = K_v v_{in}(t)$$

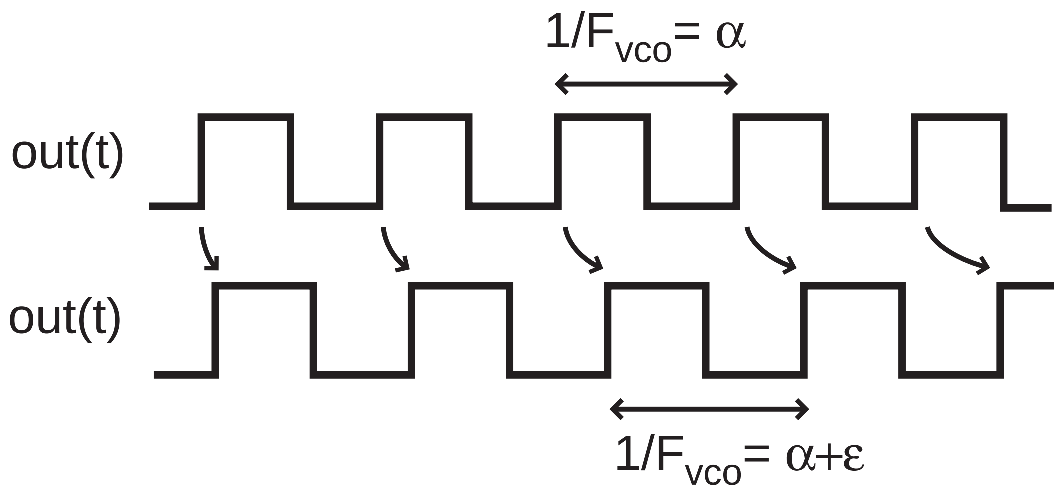
Model for Voltage to Phase Mapping of VCO

$$F_{out}(t) = K_v v_{in}(t)$$

- **Phase is more convenient than frequency for analysis**
  - **The two are related through an integral relationship**

$$\Phi_{out}(t) = \int_{-\infty}^{t} 2\pi F_{out}(\tau)d\tau = \int_{-\infty}^{t} 2\pi K_v v_{in}(\tau)d\tau$$

- **Intuition of integral relationship between frequency and phase**



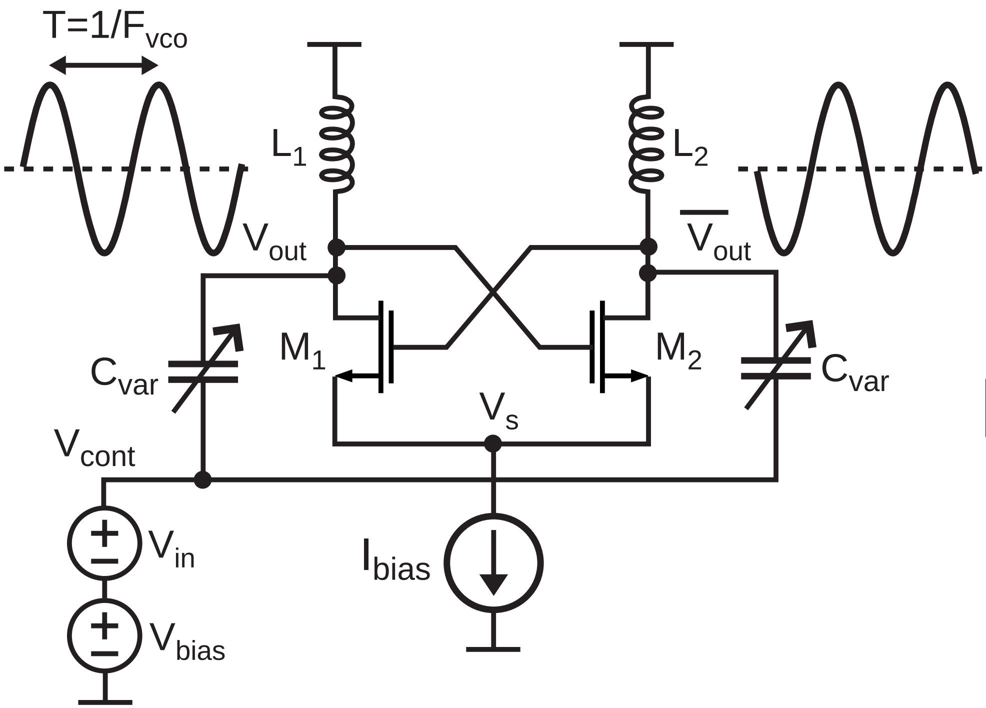
Frequency Domain Model of VCO

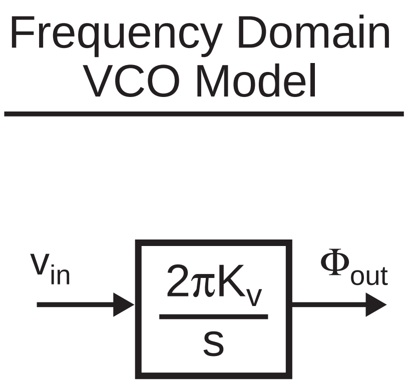
- **Take Laplace Transform of phase relationship**

$$\Phi_{out}(t) = \int_{-\infty}^{t} 2\pi K_v v_{in}(\tau)d\tau$$

$$\Rightarrow \quad \Phi_{out}(s) = 2\pi K_v v_{in}(s)$$

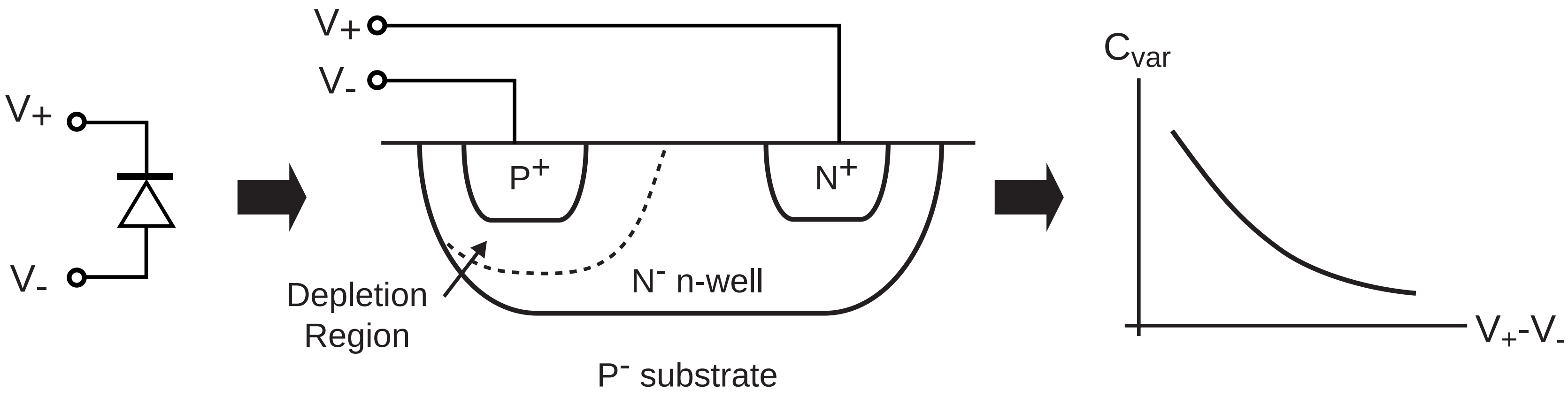
  - **Note that K_v is in units of Hz/V**





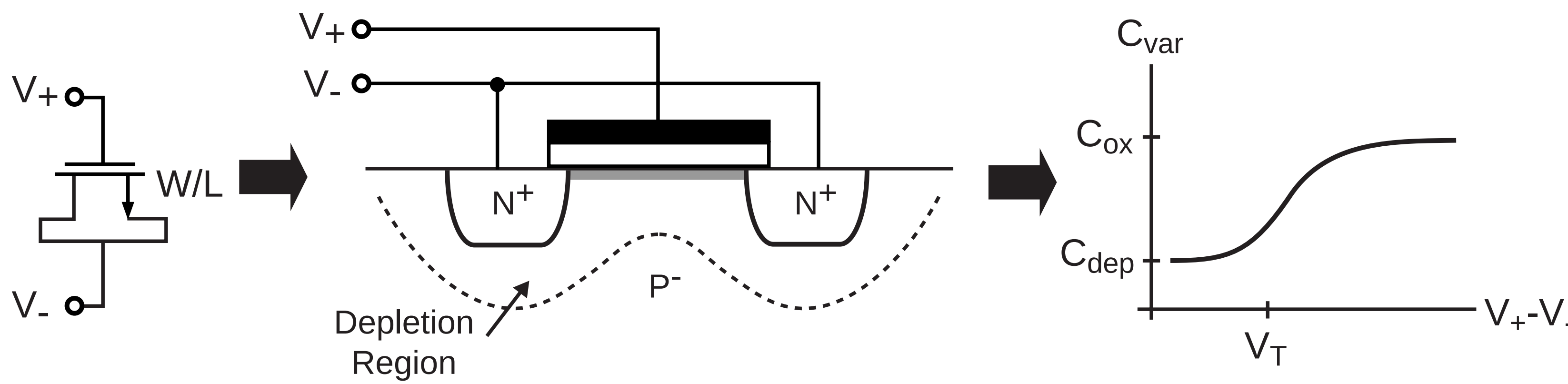
Varactor Implementation – Diode Version

- **Consists of a reverse biased diode junction**
  - **Variable capacitor formed by depletion capacitance**
  - **Capacitance drops as roughly the square root of the bias voltage**
- **Advantage – can be fully integrated in CMOS**
- **Disadvantages – low Q (often < 20), and low tuning range (§ 20%)**

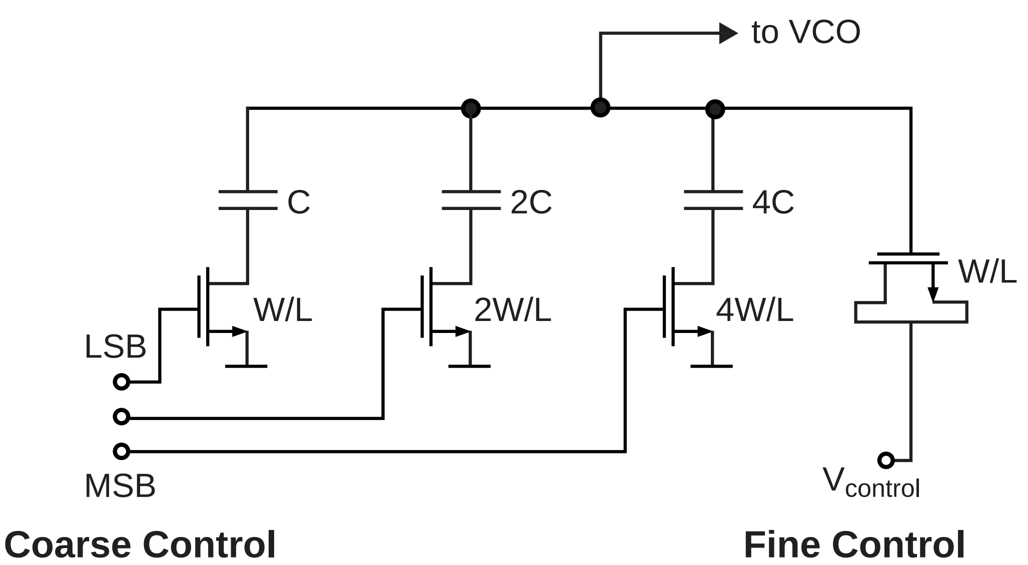


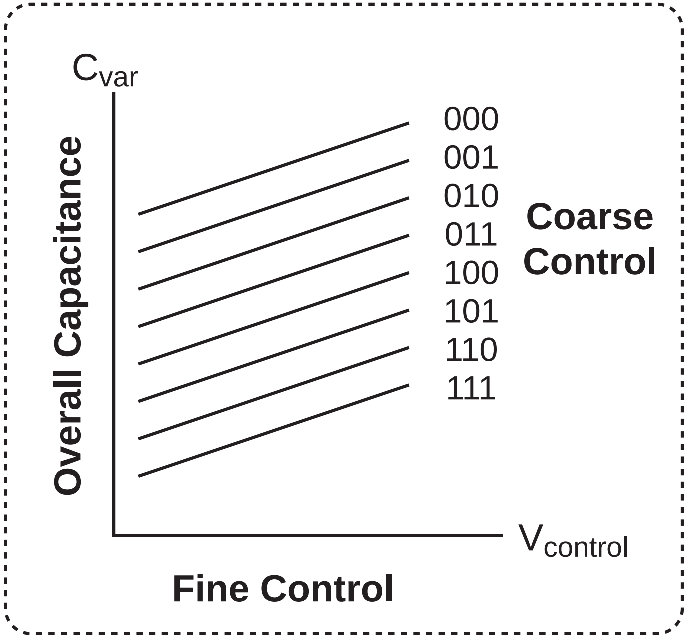
A Recently Popular Approach – The MOS Varactor

- **Consists of a MOS transistor (NMOS or PMOS) with drain and source connected together**
  - **Abrupt shift in capacitance as inversion channel forms**
- **Advantage – easily integrated in CMOS**
- **Disadvantage – Q is relatively low in the transition region**
  - **Note that large signal is applied to varactor – transition region will be swept across each VCO cycle**



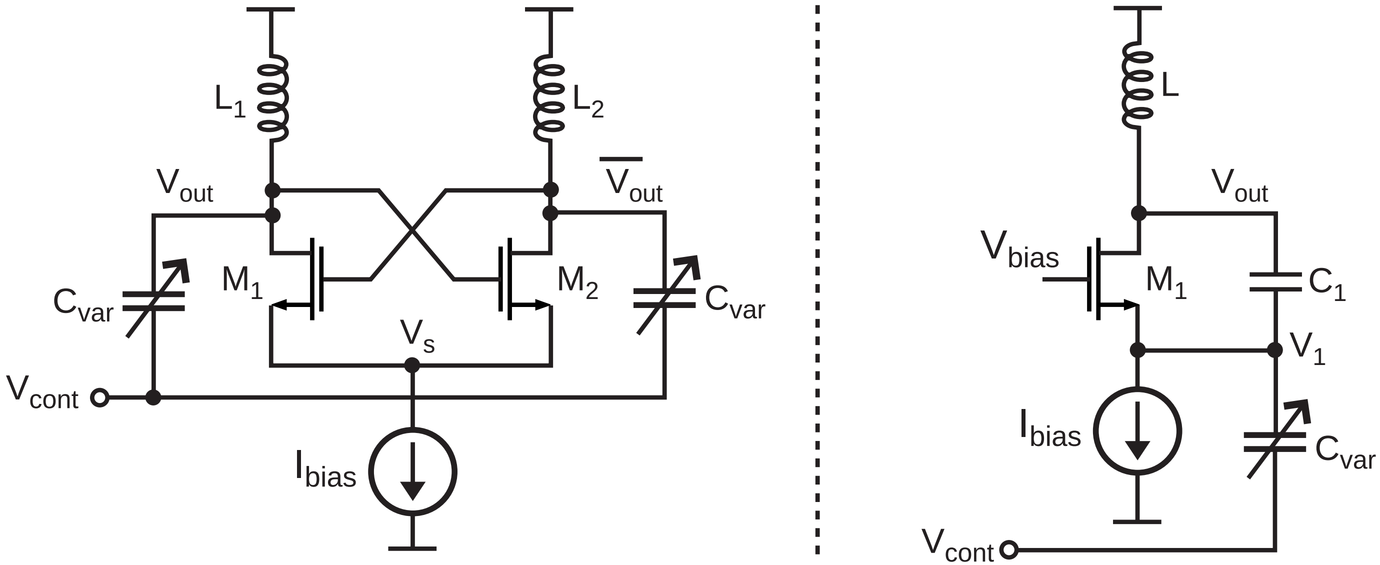
A Method To Increase Q of MOS Varactor





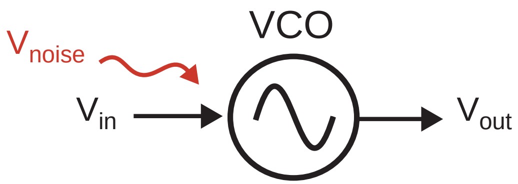
- **High Q metal caps are switched in to provide coarse tuning**
- **Low Q MOS varactor used to obtain fine tuning**
- **See Hegazi et. al., "A Filtering Technique to Lower LC Oscillator Phase Noise", JSSC, Dec 2001, pp 1921-1930**

Supply Pulling and Pushing

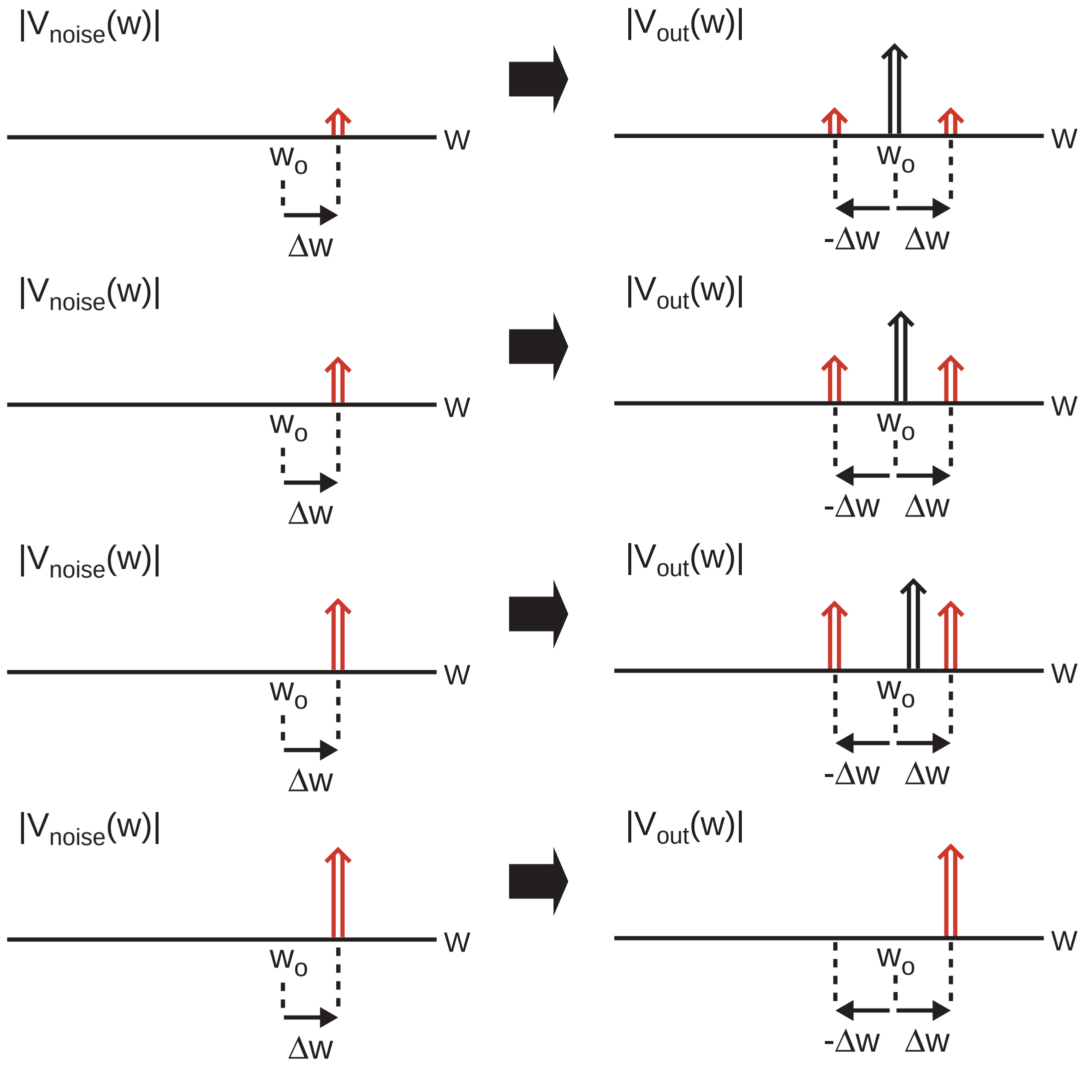


- **Supply voltage has an impact on the VCO frequency**
  - **Voltage across varactor will vary, thereby causing a shift in its capacitance**
  - **Voltage across transistor drain junctions will vary, thereby causing a shift in its depletion capacitance**
- **This problem is addressed by building a supply regulator specifically for the VCO**

Injection Locking

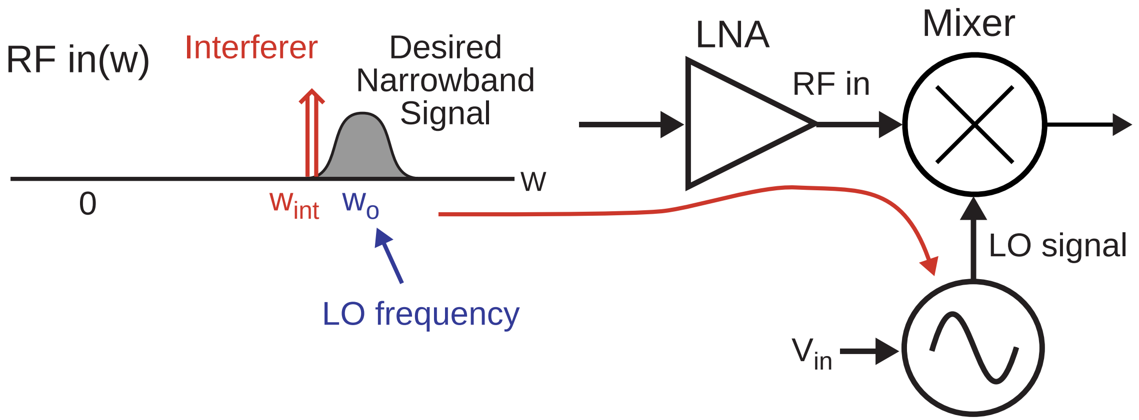


- **Noise close in frequency to VCO resonant frequency can cause VCO frequency to shift when its amplitude becomes high enough**



Example of Injection Locking

- **For homodyne systems, VCO frequency can be very close to that of interferers**



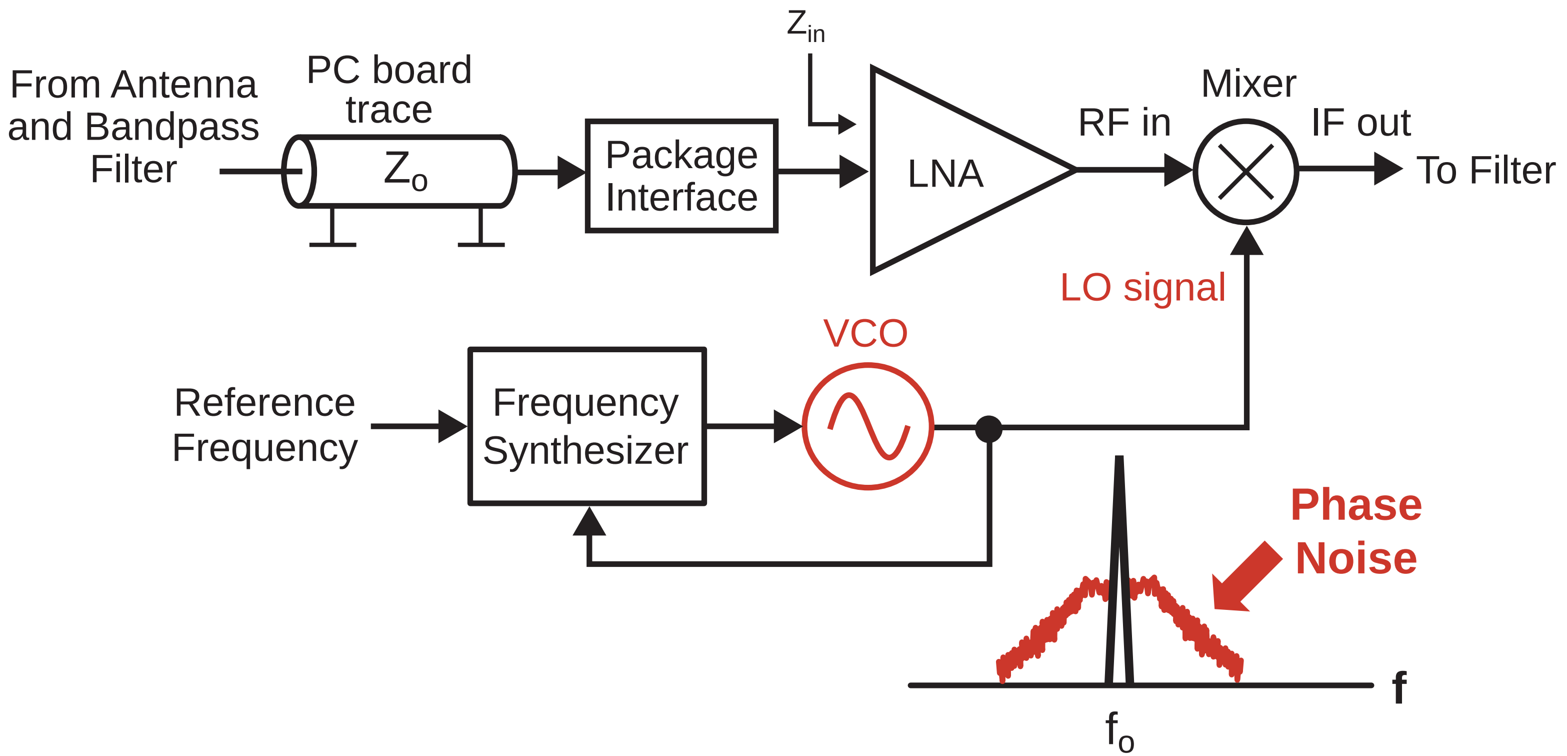
  - **Injection locking can happen if inadequate isolation from mixer RF input to LO port**
- **Follow VCO with a buffer stage with high reverse isolation to alleviate this problem**

Summary

- **Several concepts are useful for understanding LC oscillators**
  - **Barkhausen criterion**
  - **Impedance transformations**
- **Voltage-controlled oscillators incorporate a tunable element such as varactor**
  - **Increased range achieved by using switched capacitor network for coarse tuning**
    - Improves varactor Q, as well
- **Several things to watch out for**
  - **Supply pulling, injection locking, coupling**

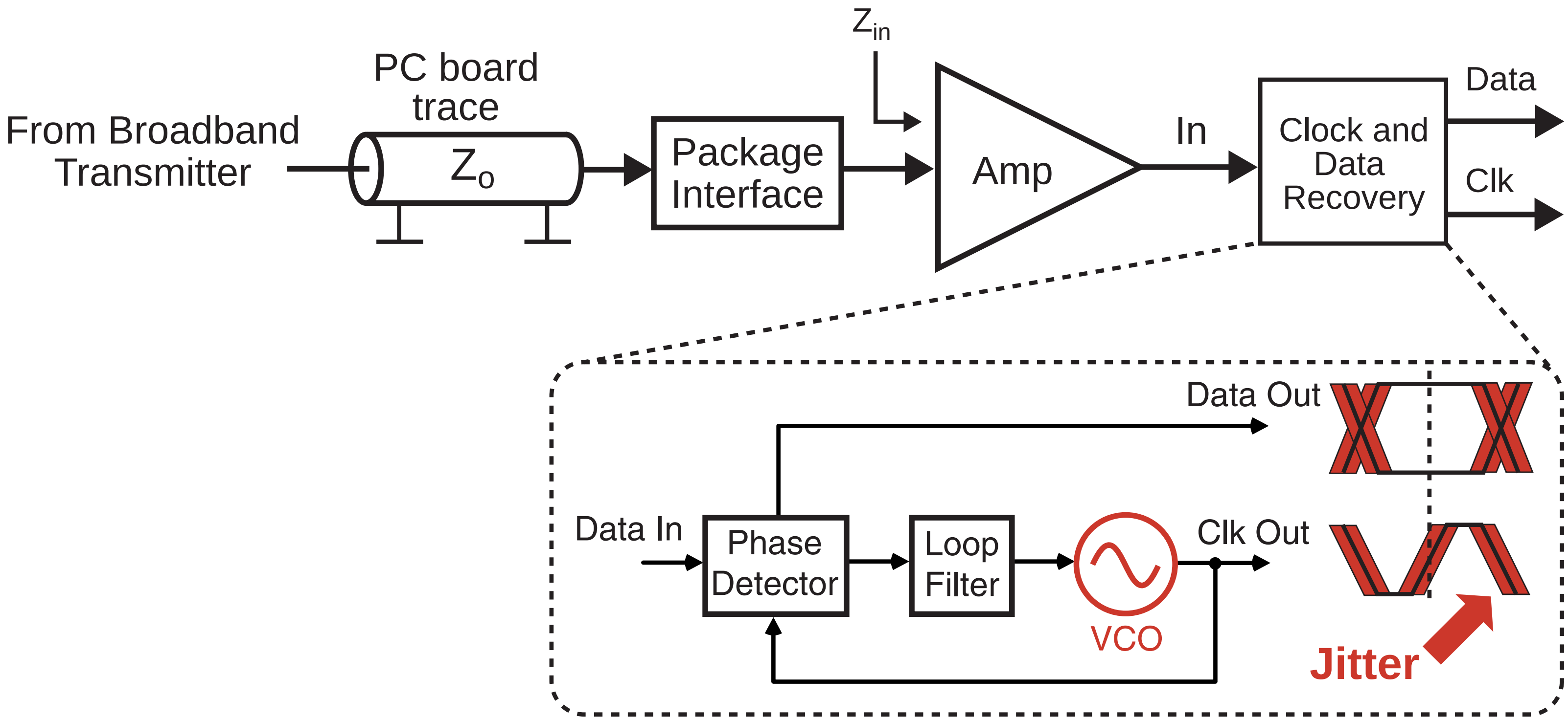
Noise in Voltage Controlled Oscillators

VCO Noise in Wireless Systems



- VCO noise has a negative impact on system performance
  - Receiver – lower sensitivity, poorer blocking performance
  - Transmitter – increased spectral emissions (output spectrum must meet a mask requirement)
- Noise is characterized in frequency domain

VCO Noise in High Speed Data Links

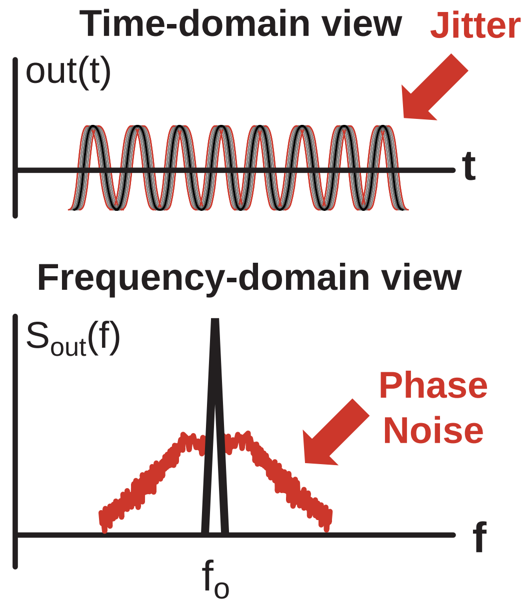


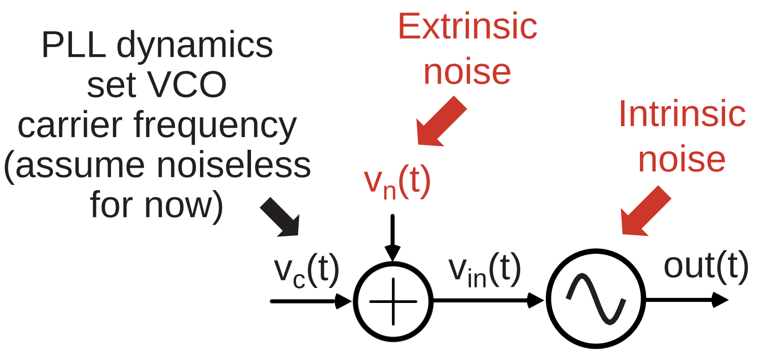
- **VCO noise also has a negative impact on data links**
  - **Receiver – increases bit error rate (BER)**
  - **Transmitter – increases jitter on data stream (transmitter must have jitter below a specified level)**
- **Noise is characterized in the time domain**

Outline of Talk

- System level view of VCO and PLL noise
- Linearized model of VCO noise
  - Noise figure
  - Equipartition theorem
  - Leeson’s formula
- Cyclo-stationary view of VCO noise
  - Hajimiri model
- Back to Leeson’s formula

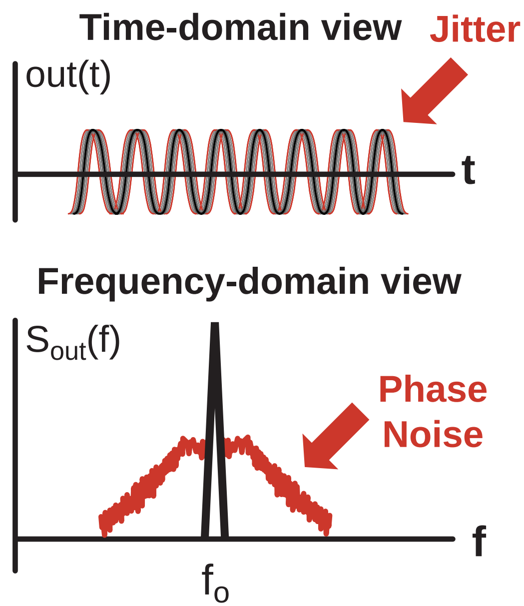
Noise Sources Impacting VCO



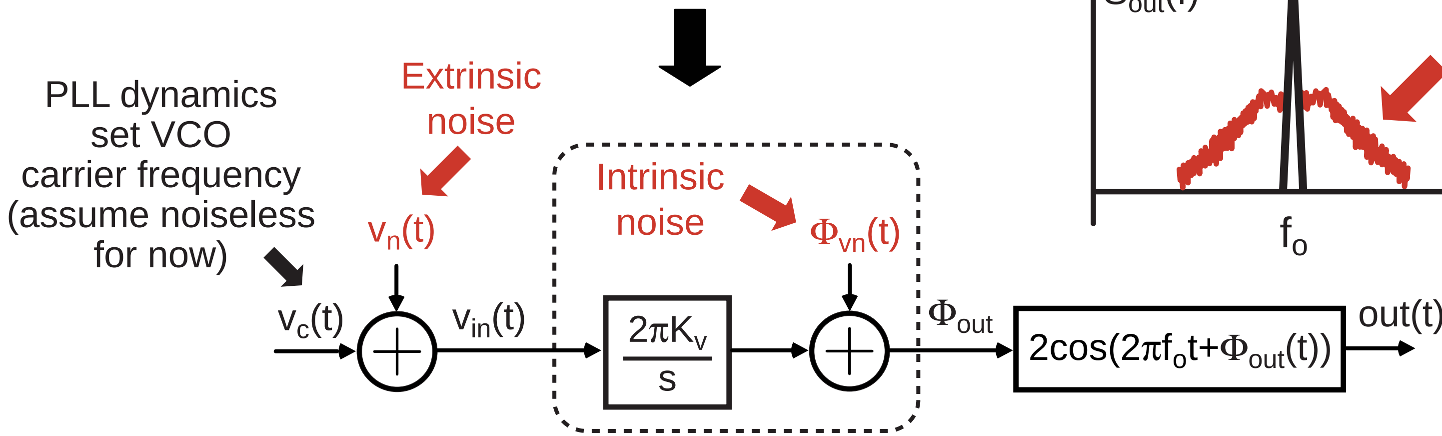


- **Extrinsic noise**
  - **Noise from other circuits (including PLL)**
- **Intrinsic noise**
  - **Noise due to the VCO circuitry**

VCO Model for Noise Analysis



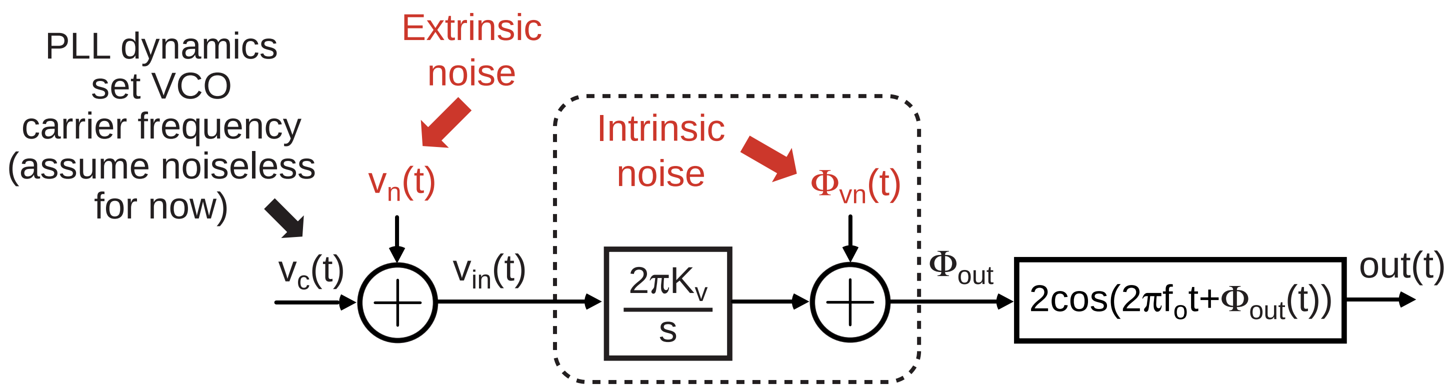
Note: K_v units are Hz/V



- **We will focus on phase noise (and its associated jitter)**
  - **Model as phase signal in output sine waveform**

$$out(t) = 2\cos(2\pi f_o t + \underline{\Phi_{out}(t)})$$

Simplified Relationship Between Φ_{out} and Output



$$out(t) = 2\cos(2\pi f_o t + \Phi_{out}(t))$$

- **Using a familiar trigonometric identity**

$$out(t) = 2\cos(2\pi f_o t)\cos(\Phi_{out}(t)) - 2\sin(2\pi f_o t)\sin(\Phi_{out}(t))$$

- **Given that the phase noise is small**

$$\cos(\Phi_{out}(t)) \approx 1, \quad \sin(\Phi_{out}(t)) \approx \Phi_{out}(t)$$

$$\Rightarrow \quad out(t) = 2\cos(2\pi f_o t) - 2\sin(2\pi f_o t)\Phi_{out}(t)$$

Calculation of Output Spectral Density

$$out(t) = 2\cos(2\pi f_o t) - 2\sin(2\pi f_o t)\Phi_{out}(t)$$

- **Calculate autocorrelation**

$$R\{out(t)\} = R\{2\cos(2\pi f_o t)\} + R\{2\sin(2\pi f_o t)\} \cdot R\{\Phi_{out}(t)\}$$

- **Take Fourier transform to get spectrum**

$$S_{out}(f) = S_{sin}(f) + S_{sin}(f) * S_{\Phi_{out}}$$

  - **Note that * symbol corresponds to convolution**
- **In general, phase spectral density can be placed into one of two categories**
  - **Phase noise – Φ_{out}(t) is non-periodic**
  - **Spurious noise - Φ_{out}(t) is periodic**

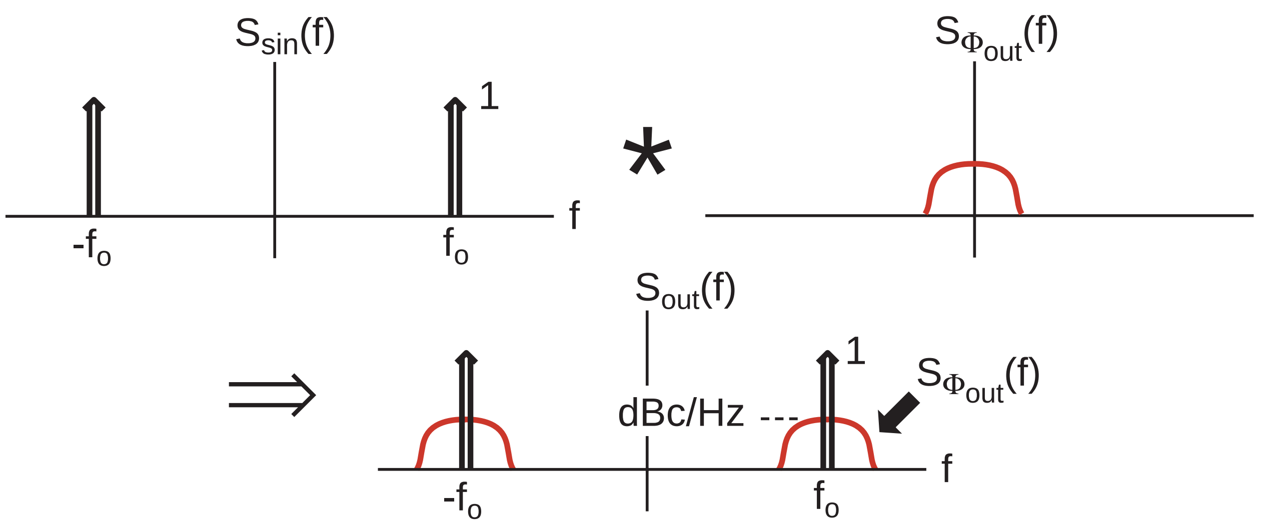
Output Spectrum with Phase Noise

- Suppose input noise to VCO ($v_n(t)$) is bandlimited, non-periodic noise with spectrum $S_{vn}(f)$
  - In practice, derive phase spectrum as

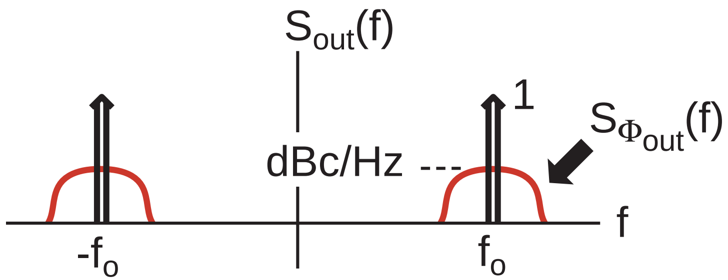
$$S_{\Phi_{out}}(f) = \left(\frac{K_v}{f}\right)^2 S_{v_n}(f)$$

- Resulting output spectrum

$$S_{out}(f) = S_{sin}(f) + S_{sin}(f) * S_{\Phi_{out}}$$



Measurement of Phase Noise in dBc/Hz



- **Definition of L(f)**

$$L(f) = 10\log\left(\frac{\text{Spectral density of noise}}{\text{Power of carrier}}\right)$$

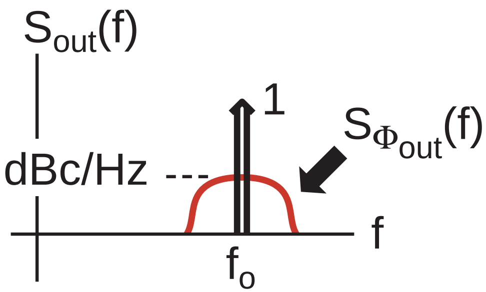
  - **Units are dBc/Hz**

- **For this case**

$$L(f) = 10\log\left(\frac{2S_{\Phi_{out}}(f)}{2}\right) = 10\log(S_{\Phi_{out}}(f))$$

  - **Valid when $\Phi_{out}(t)$ is small in deviation (i.e., when carrier is not modulated, as currently assumed)**

Single-Sided Version



- **Definition of L(f) remains the same**

$$L(f) = 10\log\left(\frac{\text{Spectral density of noise}}{\text{Power of carrier}}\right)$$

  - **Units are dBc/Hz**

- **For this case**

$$L(f) = 10\log\left(\frac{S_{\Phi_{out}}(f)}{1}\right) = 10\log(S_{\Phi_{out}}(f))$$

  - **So, we can work with either one-sided or two-sided spectral densities since L(f) is set by *ratio* of noise density to carrier power**

Output Spectrum with Spurious Noise

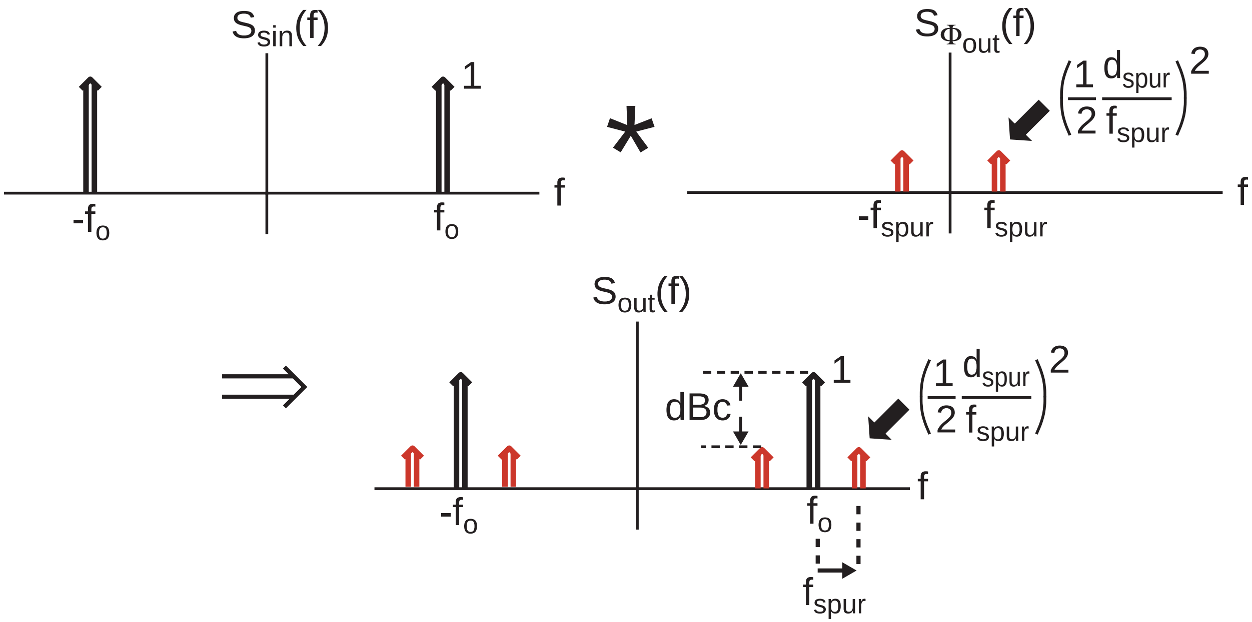
- **Suppose input noise to VCO is**

$$v_n(t) = \frac{d_{spur}}{K_v}\cos(2\pi f_{spur} t)$$

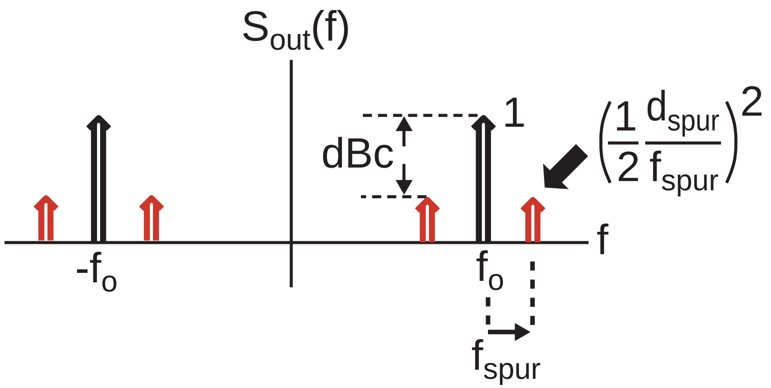
$$\Rightarrow \quad \Phi_{out}(t) = 2\pi K_v \int v_n(t)dt = \frac{d_{spur}}{f_{spur}}\sin(2\pi f_{spur} t)$$

- **Resulting output spectrum**

$$S_{out}(f) = S_{sin}(f) + S_{sin}(f) * S_{\Phi_{out}}$$



Measurement of Spurious Noise in dBc



- **Definition of dBc**

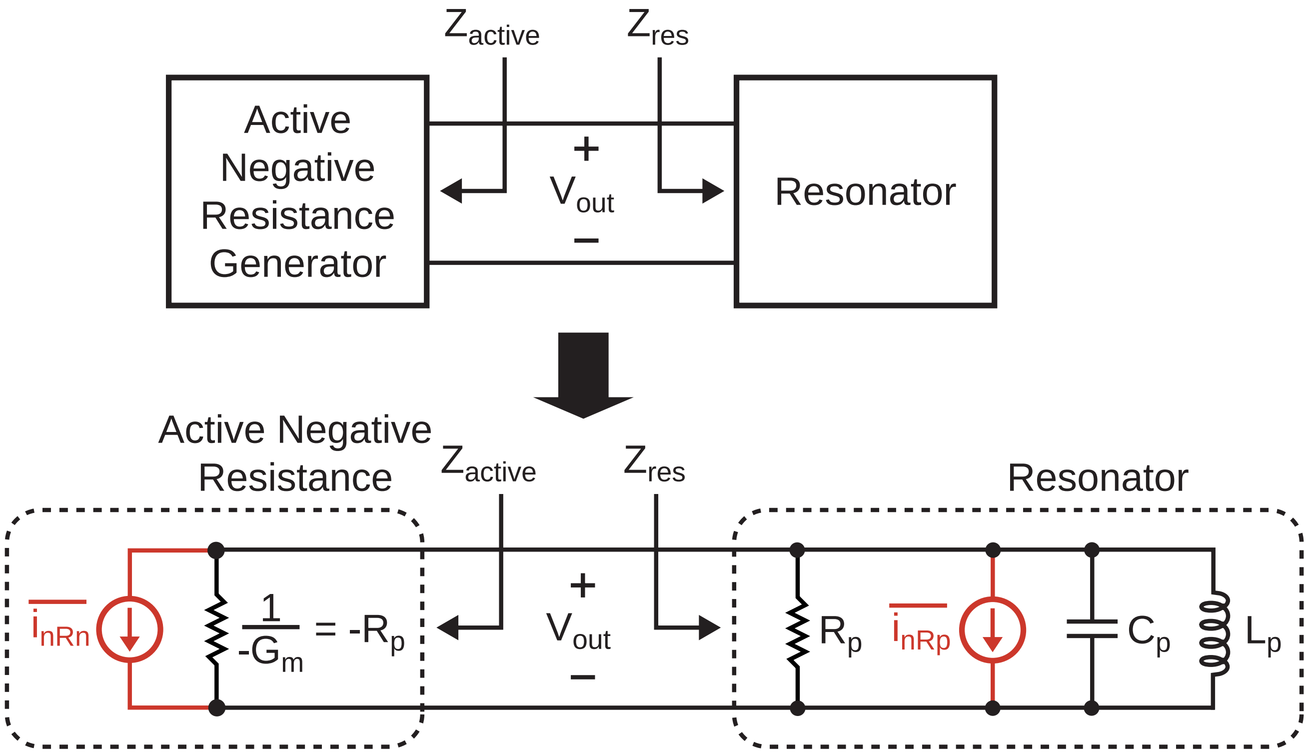
$$10\log\left(\frac{\text{Power of tone}}{\text{Power of carrier}}\right)$$

  - **We are assuming double sided spectra, so integrate over positive and negative frequencies to get power**
    - Either single or double-sided spectra can be used in practice

- **For this case**

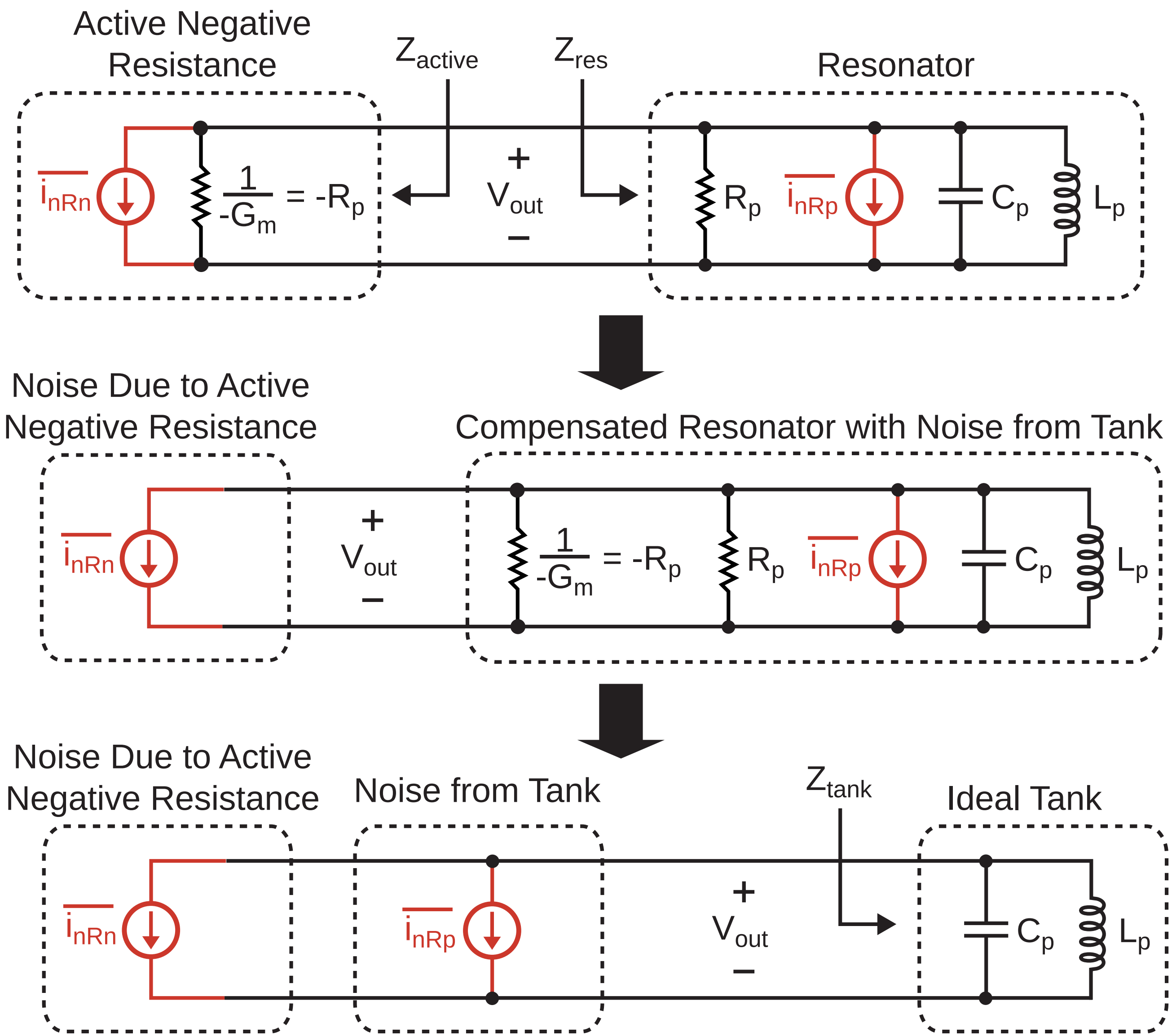
$$10\log\left(\frac{2(\frac{d_{spur}}{2f_{spur}})^2}{2}\right) = 20\log\left(\frac{d_{spur}}{2f_{spur}}\right)\ dBc$$

Calculation of Intrinsic Phase Noise in Oscillators

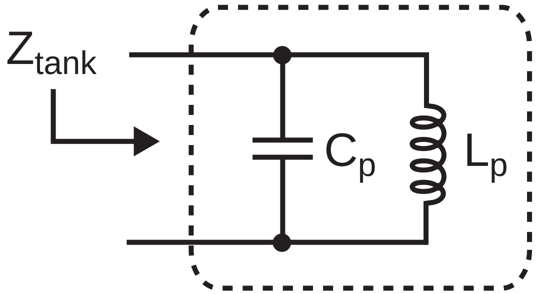


- Noise sources in oscillators are put in two categories
  - Noise due to tank loss
  - Noise due to active negative resistance
- We want to determine how these noise sources influence the phase noise of the oscillator

Equivalent Model for Noise Calculations



Calculate Impedance Across Ideal LC Tank Circuit



$$Z_{tank}(w) = \frac{1}{jwC_p} || jwL_p = \frac{jwL_p}{1 - w^2 L_p C_p}$$

- **Calculate input impedance about resonance**

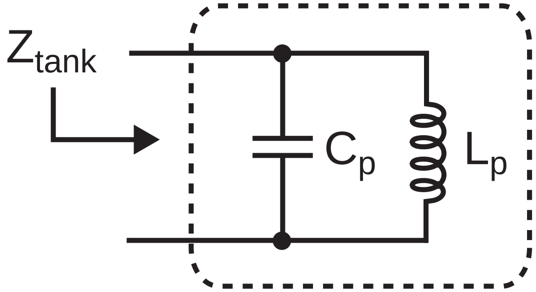
$$\text{Consider } w = w_o + \Delta w, \text{ where } w_o = \frac{1}{\sqrt{L_p C_p}}$$

$$Z_{tank}(\Delta w) = \frac{j(w_o + \Delta w)L_p}{1 - (w_o + \Delta w)^2 L_p C_p}$$

$$= \frac{j(w_o + \Delta w)L_p}{\underbrace{1 - w_o^2 L_p C_p}_{=0} - 2\Delta w(w_o L_p C_p) - \underbrace{\Delta w^2 L_p C_p}_{\text{negligible}}} \approx \frac{j(w_o + \Delta w)L_p}{-2\Delta w(w_o L_p C_p)}$$

$$\Rightarrow Z_{tank}(\Delta w) \approx \frac{jw_o L_p}{-2\Delta w(w_o L_p C_p)} = \boxed{-\frac{j}{2}\frac{1}{w_o C_p}\left(\frac{w_o}{\Delta w}\right)}$$

A Convenient Parameterization of LC Tank Impedance



$$Z_{tank}(\Delta w) \approx -\frac{j}{2}\frac{1}{w_o C_p}\left(\frac{w_o}{\Delta w}\right)$$

- Actual tank has loss that is modeled with R_p
  - Define Q according to actual tank

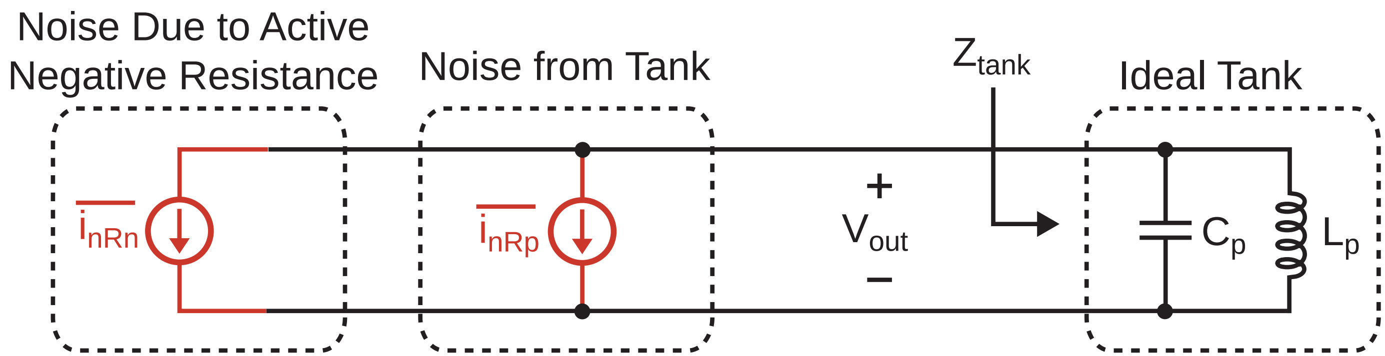
$$Q = R_p w_o C_p \quad \Rightarrow \quad \frac{1}{w_o C_p} = \frac{R_p}{Q}$$

- Parameterize ideal tank impedance in terms of Q of actual tank

$$Z_{tank}(\Delta w) \approx -\frac{j}{2}\frac{R_p}{Q}\left(\frac{w_o}{\Delta w}\right)$$

$$\Rightarrow \quad |Z_{tank}(\Delta f)|^2 \approx \boxed{\left(\frac{R_p}{2Q}\frac{f_o}{\Delta f}\right)^2}$$

Overall Noise Output Spectral Density



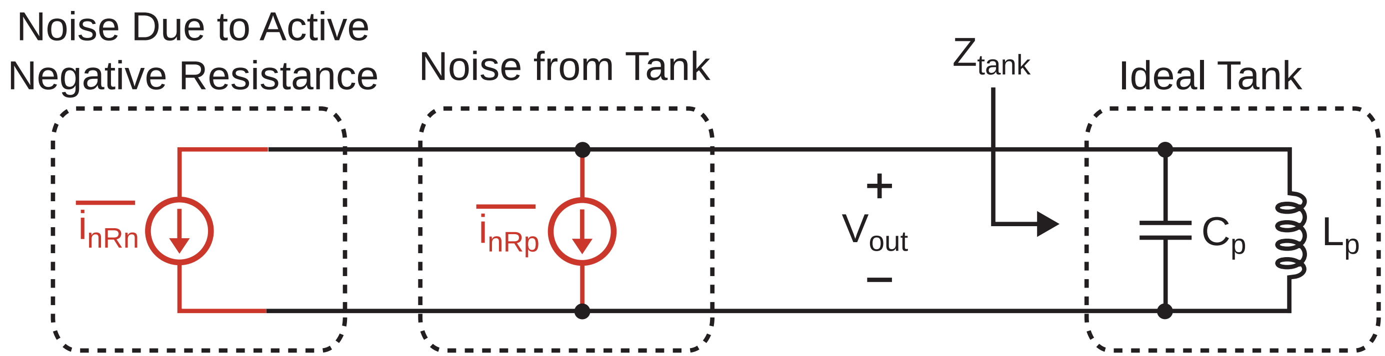
- **Assume noise from active negative resistance element and tank are uncorrelated**

$$\frac{\overline{v_{out}^2}}{\Delta f} = \left(\frac{\overline{i_{nRp}^2}}{\Delta f} + \frac{\overline{i_{nRn}^2}}{\Delta f}\right) |Z_{tank}(\Delta f)|^2$$

$$= \frac{\overline{i_{nRp}^2}}{\Delta f} \left(1 + \frac{\overline{i_{nRn}^2}}{\Delta f} \Big/ \frac{\overline{i_{nRp}^2}}{\Delta f}\right) |Z_{tank}(\Delta f)|^2$$

  - **Note that the above expression represents total noise that impacts both amplitude and phase of oscillator output**

Parameterize Noise Output Spectral Density



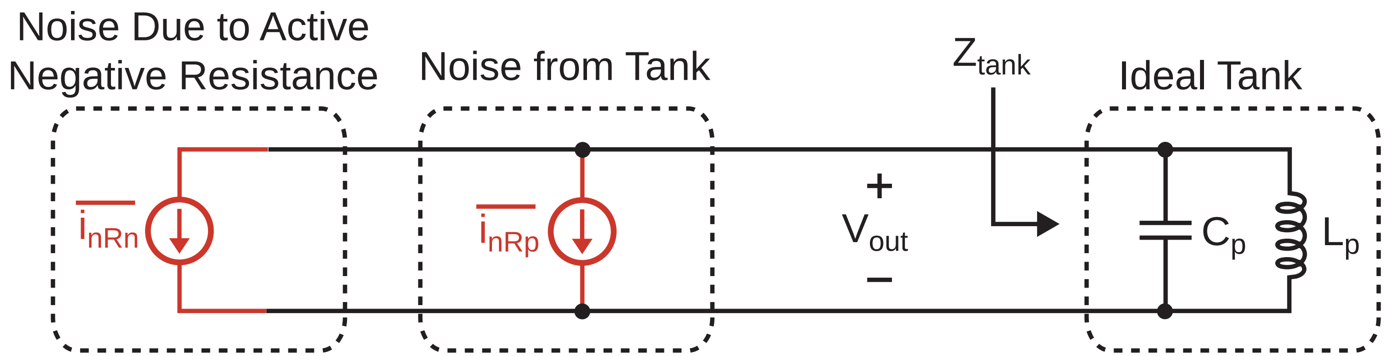
- **From previous slide**

$$\frac{\overline{v_{out}^2}}{\Delta f} = \frac{\overline{i_{nRp}^2}}{\Delta f}\underbrace{\left(1 + \frac{\overline{i_{nRn}^2}}{\Delta f} \Big/ \frac{\overline{i_{nRp}^2}}{\Delta f}\right)}_{\mathbf{F(\Delta f)}} |Z_{tank}(\Delta f)|^2$$

- **F(Δf) is defined as**

$$F(\Delta f) = \frac{\text{total noise in tank at frequency } \Delta f}{\text{noise in tank due to tank loss at frequency } \Delta f}$$

Fill in Expressions



- **Noise from tank is due to resistor R_p**

$$\frac{\overline{i^2_{nRp}}}{\Delta f} = 4kT\frac{1}{R_p} \quad \text{(single-sided spectrum)}$$

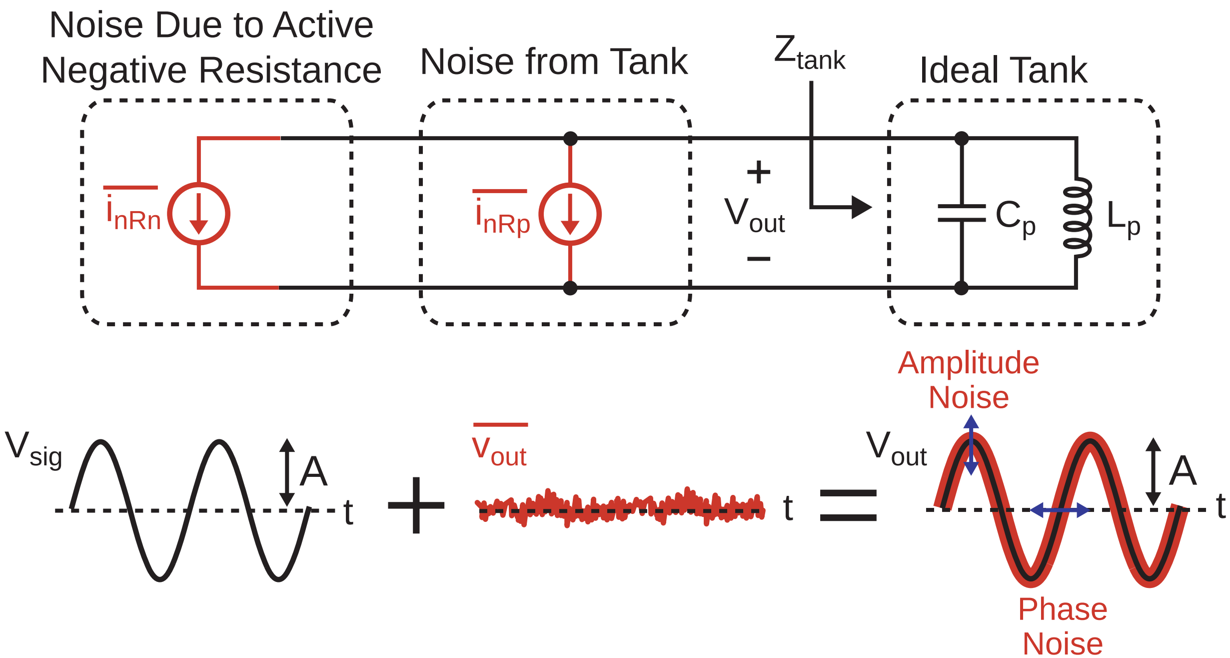
- **$Z_{tank}(\Delta f)$ found previously**

$$|Z_{tank}(\Delta f)|^2 \approx \left(\frac{R_p}{2Q}\frac{f_o}{\Delta f}\right)^2$$

- **Output noise spectral density expression (single-sided)**

$$\frac{\overline{v^2_{out}}}{\Delta f} = 4kT\frac{1}{R_p}F(\Delta f)\left(\frac{R_p}{2Q}\frac{f_o}{\Delta f}\right)^2 = \boxed{4kTF(\Delta f)R_p\left(\frac{1}{2Q}\frac{f_o}{\Delta f}\right)^2}$$

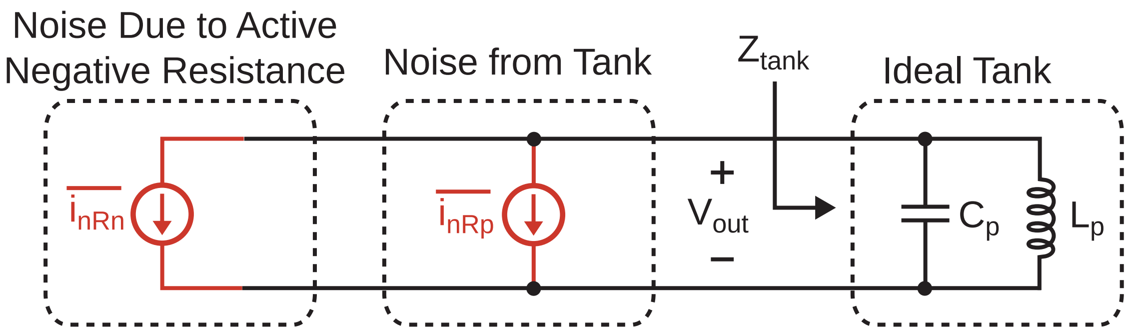
Separation into Amplitude and Phase Noise



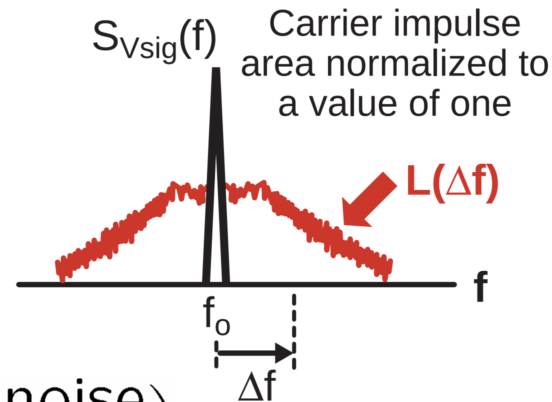
- **Equipartition theorem states that noise impact splits evenly between amplitude and phase for V_{sig} being a sine wave**
  - **Amplitude variations suppressed by feedback in oscillator**

$$\Rightarrow \quad \frac{\overline{v_{out}^2}}{\Delta f}\Bigg|_{\text{phase}} = \boxed{2kTF(\Delta f)R_p\left(\frac{1}{2Q}\frac{f_o}{\Delta f}\right)^2} \quad \text{(single-sided)}$$

Output Phase Noise Spectrum (Leeson's Formula)



Output Spectrum



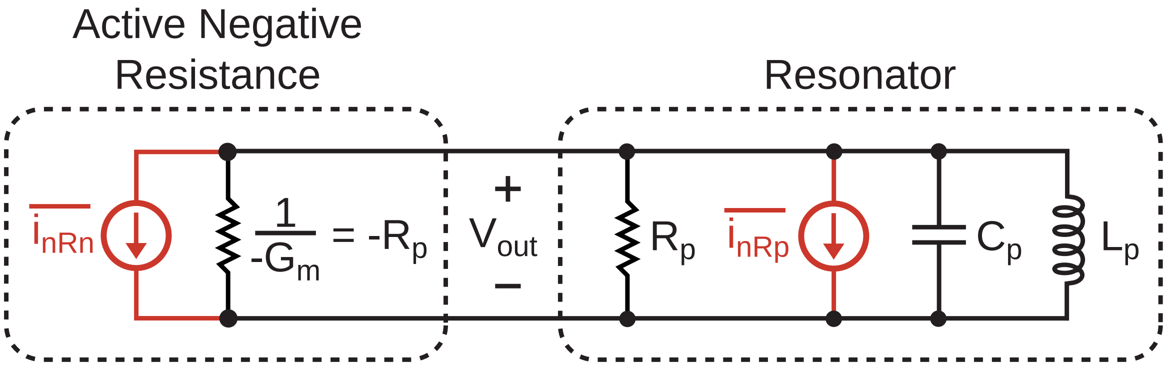
$$L(\Delta f) = 10\log\left(\frac{\text{Spectral density of noise}}{\text{Power of carrier}}\right)$$

- **All power calculations are referenced to the tank loss resistance, R_p**

$$P_{sig} = \frac{V_{sig,rms}^2}{R_p} = \frac{(A/\sqrt{2})^2}{R_p}, \quad S_{noise}(\Delta f) = \frac{1}{R_p}\frac{\overline{v_{out}^2}}{\Delta f}$$

$$L(\Delta f) = 10\log\left(\frac{S_{noise}(\Delta f)}{P_{sig}}\right) = 10\log\left(\frac{2kTF(\Delta f)}{P_{sig}}\left(\frac{1}{2Q}\frac{f_o}{\Delta f}\right)^2\right)$$

Example: Active Noise Same as Tank Noise

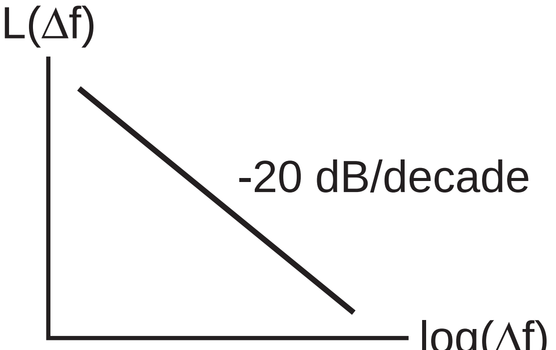


- **Noise factor for oscillator in this case is**

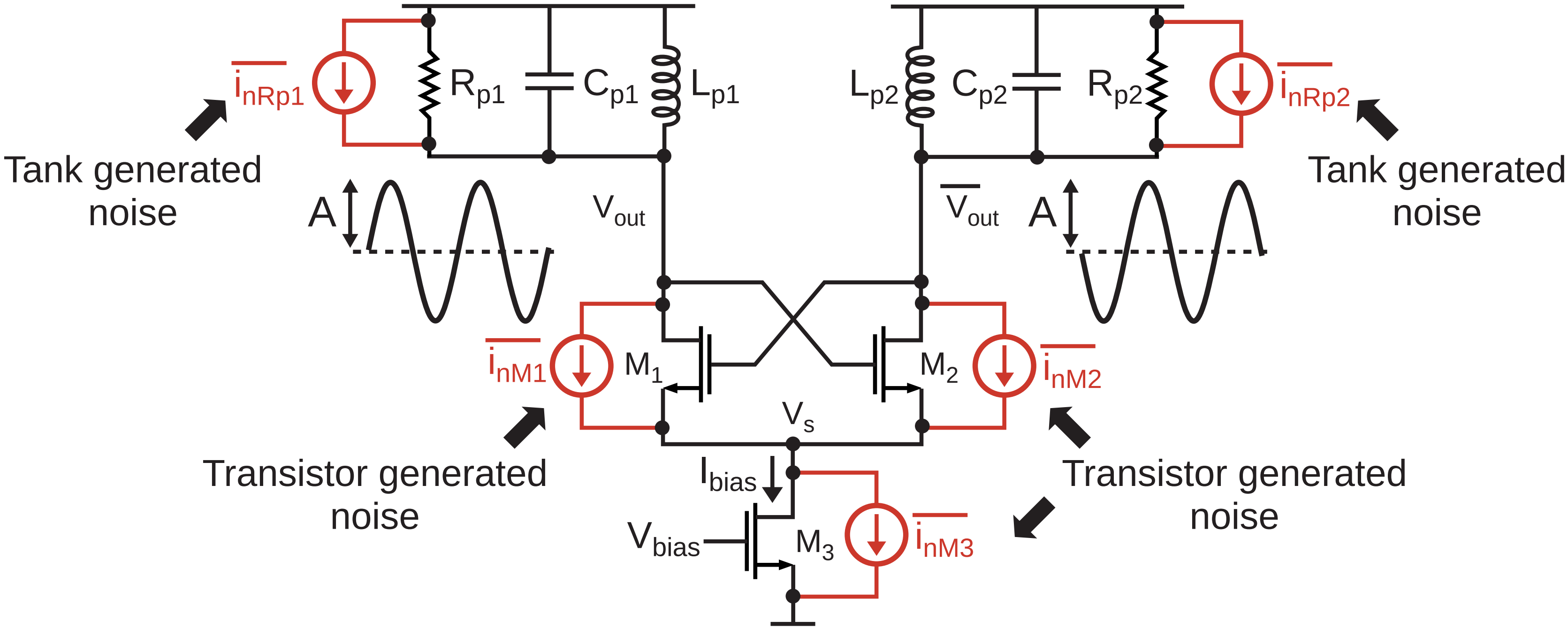
$$F(\Delta f) = 1 + \frac{\overline{i^2_{nRn}}}{\Delta f} \Big/ \frac{\overline{i^2_{nRp}}}{\Delta f} = 2$$

- **Resulting phase noise**

$$L(\Delta f) = 10\log\left(\frac{4kT}{P_{sig}}\left(\frac{1}{2Q}\frac{f_o}{\Delta f}\right)^2\right)$$

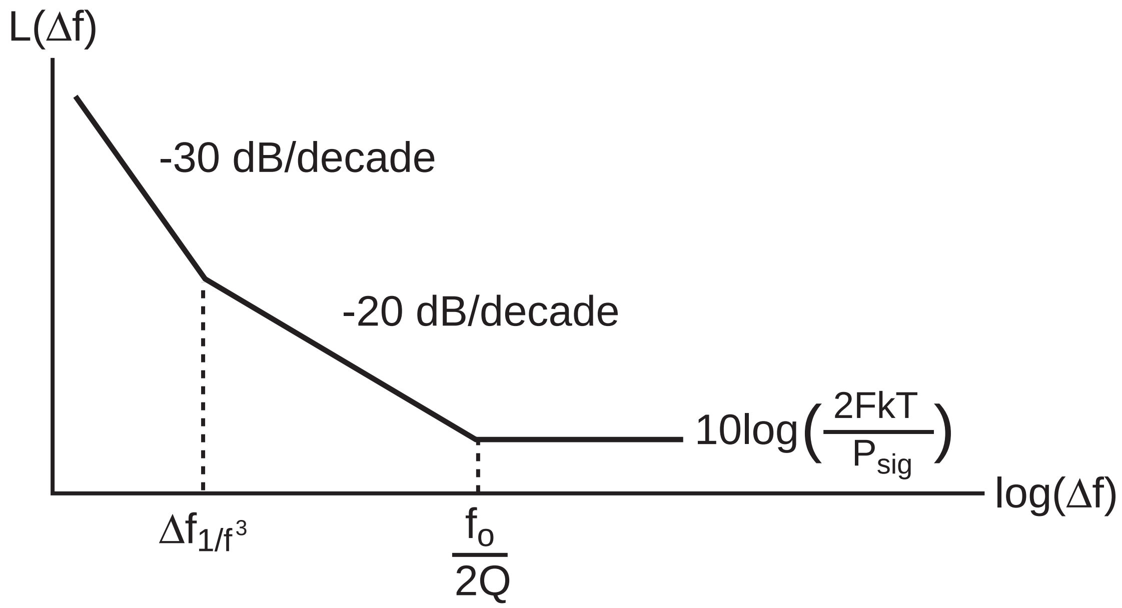


The Actual Situation is Much More Complicated



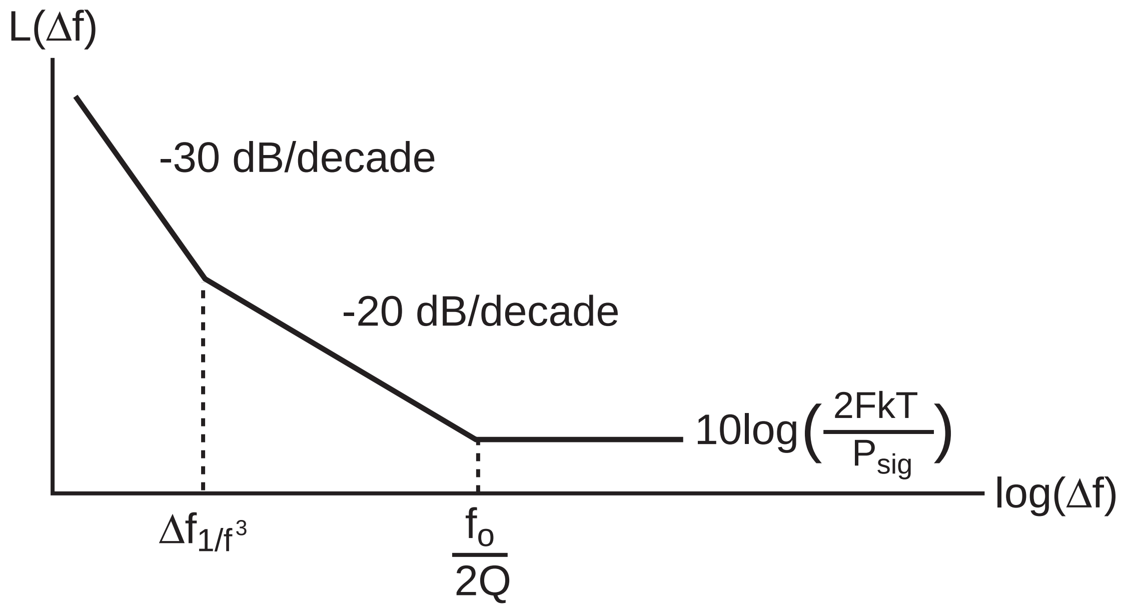
- **Impact of tank generated noise easy to assess**
- **Impact of transistor generated noise is complicated**
  - **Noise from M_1 and M_2 is modulated on and off**
  - **Noise from M_3 is modulated before influencing V_{out}**
  - **Transistors have 1/f noise**
- **Also, transistors can degrade Q of tank**

Phase Noise of A Practical Oscillator



- **Phase noise drops at -20 dB/decade over a wide frequency range, but deviates from this at:**
  - **Low frequencies – slope increases (often -30 dB/decade)**
  - **High frequencies – slope flattens out (oscillator tank does not filter all noise sources)**
- **Frequency breakpoints and magnitude scaling are not readily predicted by the analysis approach taken so far**

Phase Noise of A Practical Oscillator

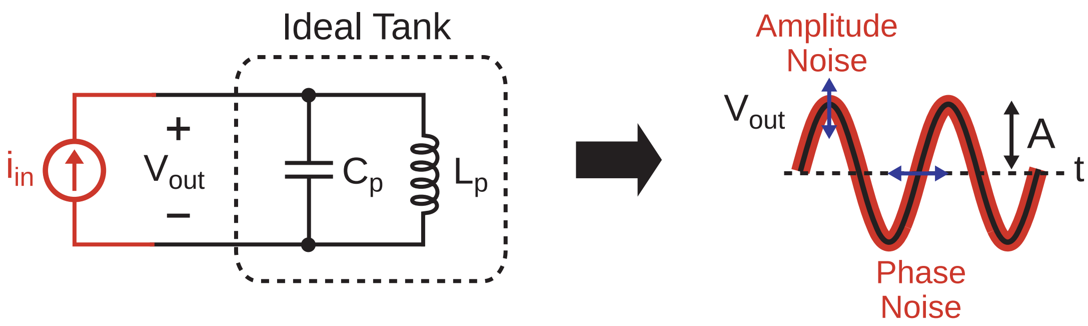


- **Leeson proposed an ad hoc modification of the phase noise expression to capture the above noise profile**

$$L(\Delta f) = 10\log\left(\frac{2FkT}{P_{sig}}\left(1+\left(\frac{1}{2Q}\frac{f_o}{\Delta f}\right)^2\right)\left(1+\frac{\Delta f_{1/f^3}}{|\Delta f|}\right)\right)$$

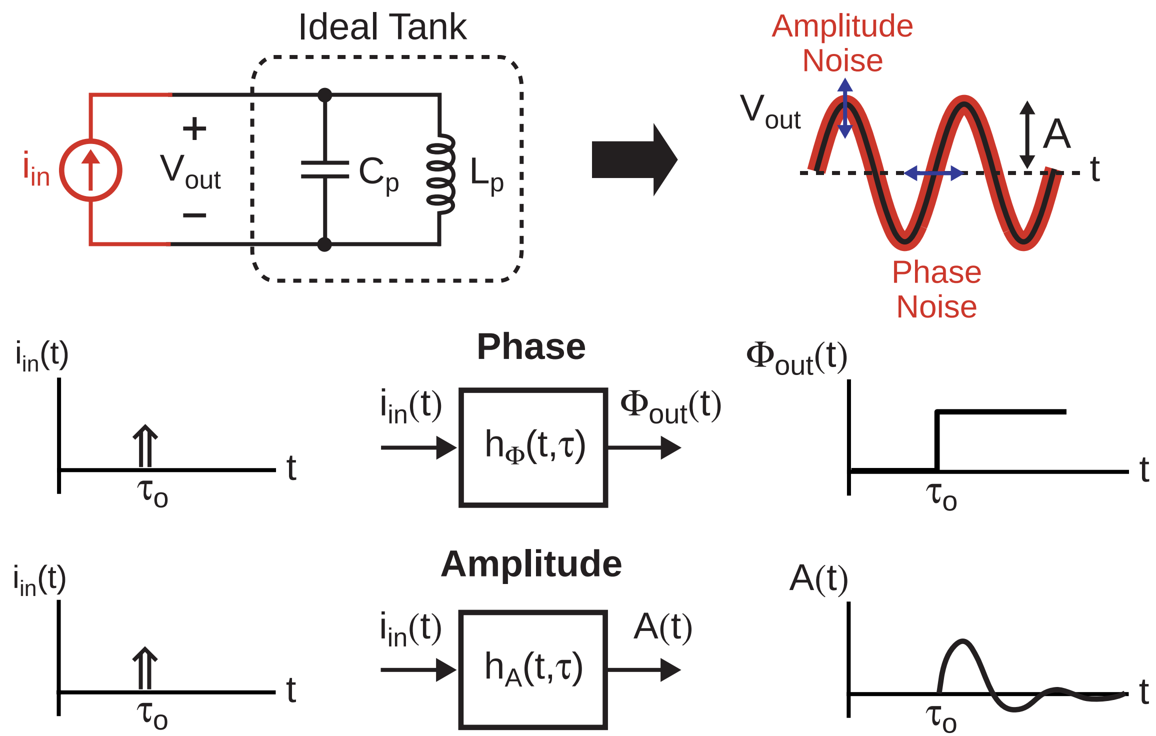
  - **Note: he assumed that F(Δf) was constant over frequency**

A More Sophisticated Analysis Method



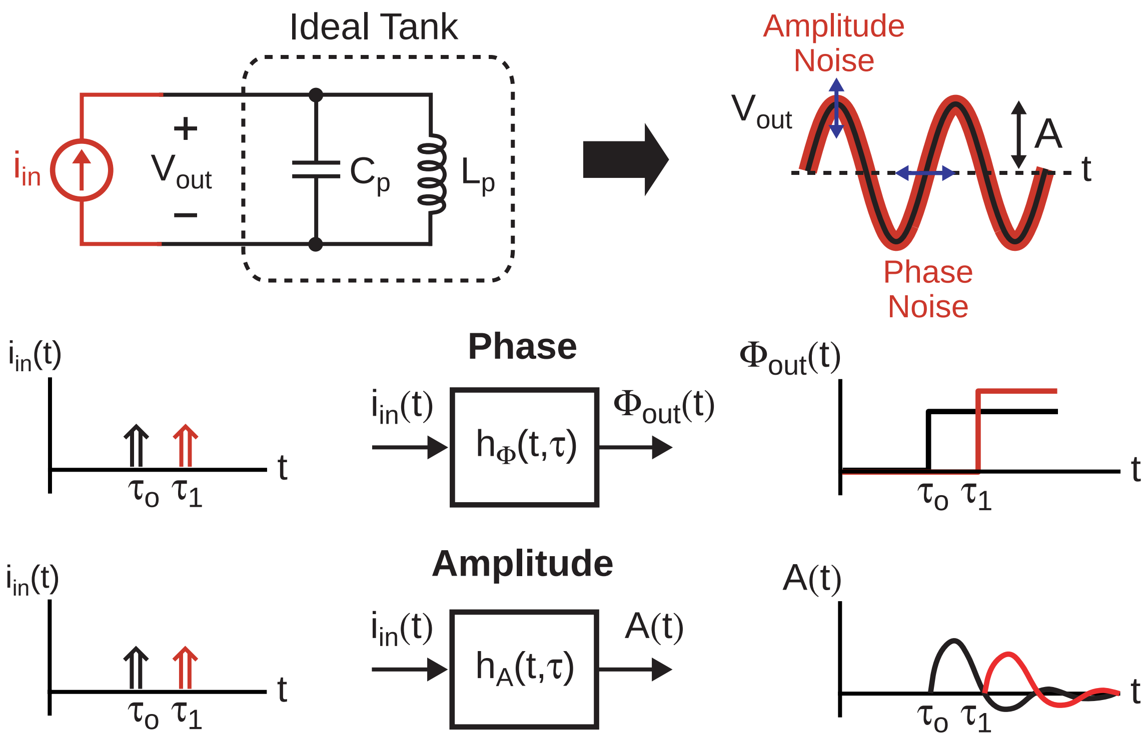
- **Our concern is what happens when noise current produces a voltage across the tank**
  - **Such voltage deviations give rise to both amplitude and phase noise**
  - **Amplitude noise is suppressed through feedback (or by amplitude limiting in following buffer stages)**
    - Our main concern is phase noise
- **We argued that impact of noise divides equally between amplitude and phase for sine wave outputs**
  - **What happens when we have a non-sine wave output?**

Modeling of Phase and Amplitude Perturbations



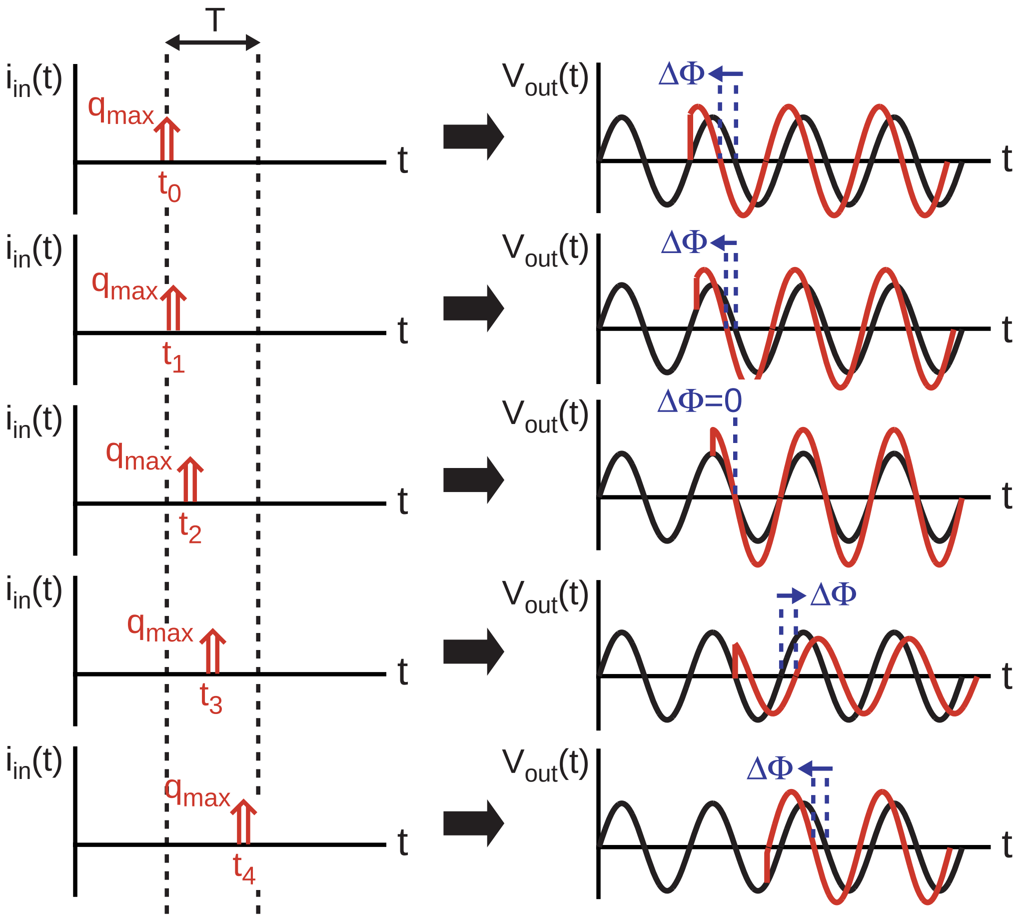
- **Characterize impact of current noise on amplitude and phase through their associated impulse responses**
  - **Phase deviations are accumulated**
  - **Amplitude deviations are suppressed**

Impact of Noise Current is Time-Varying



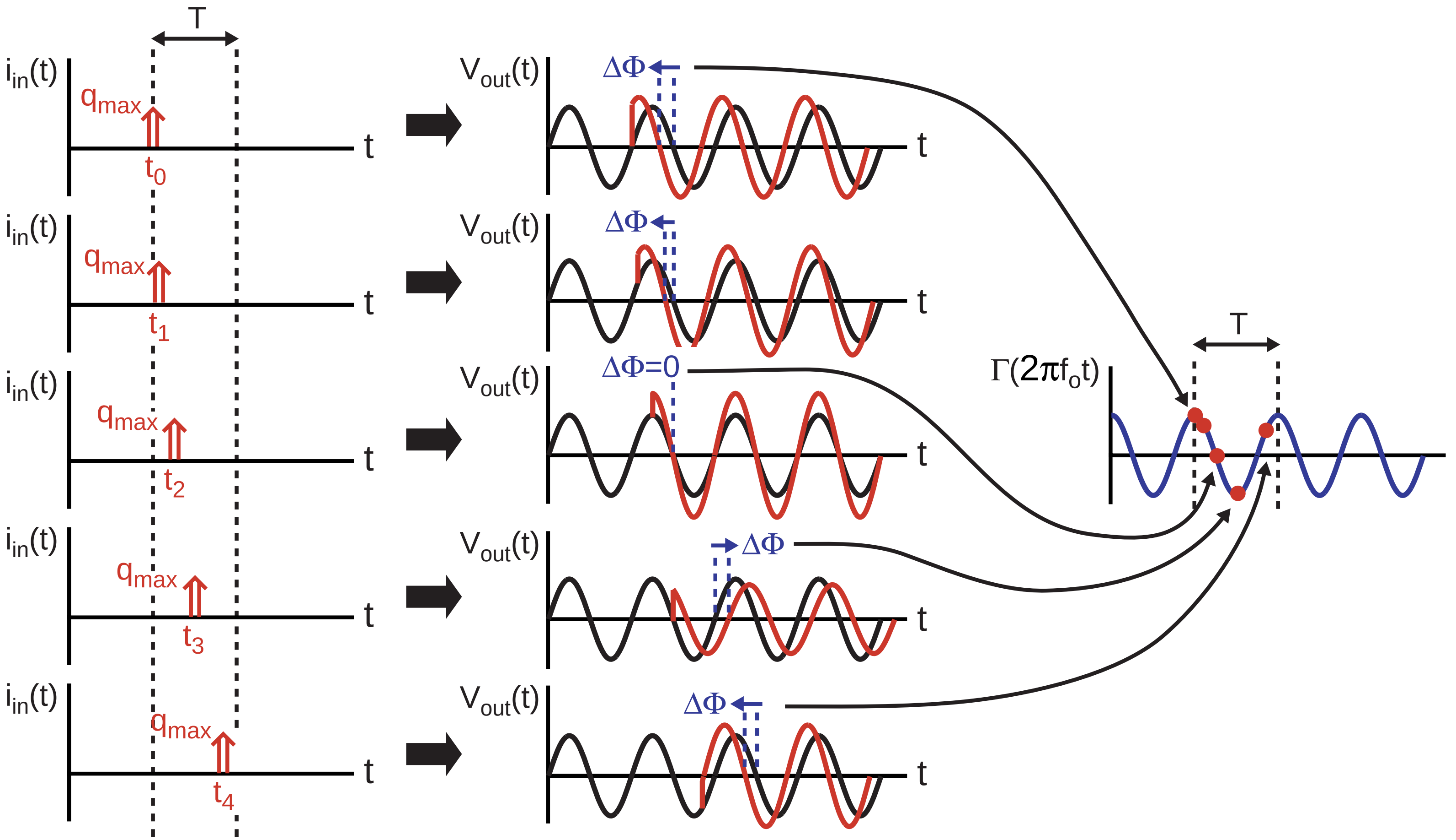
- **If we vary the time at which the current impulse is injected, its impact on phase and amplitude changes**
  - **Need a time-varying model**

Illustration of Time-Varying Impact of Noise on Phase



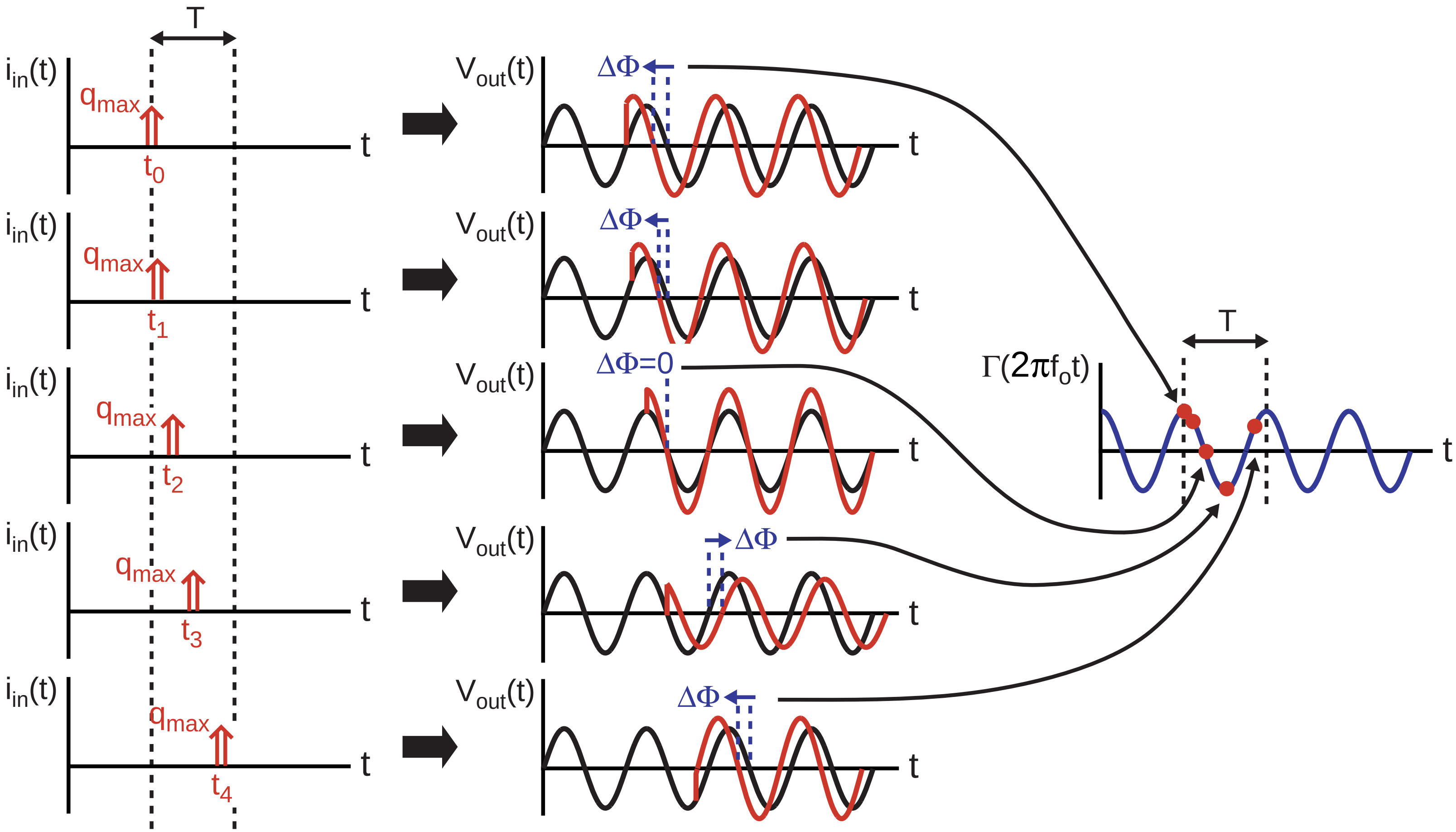
- **High impact on phase when impulse occurs close to the zero crossing of the VCO output**
- **Low impact on phase when impulse occurs at peak of output**

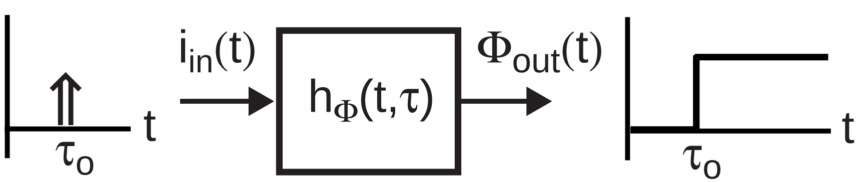
Define Impulse Sensitivity Function (ISF) – $\Gamma(2\pi f_o t)$



- **ISF constructed by calculating phase deviations as impulse position is varied**
  - **Observe that it is periodic with same period as VCO output**

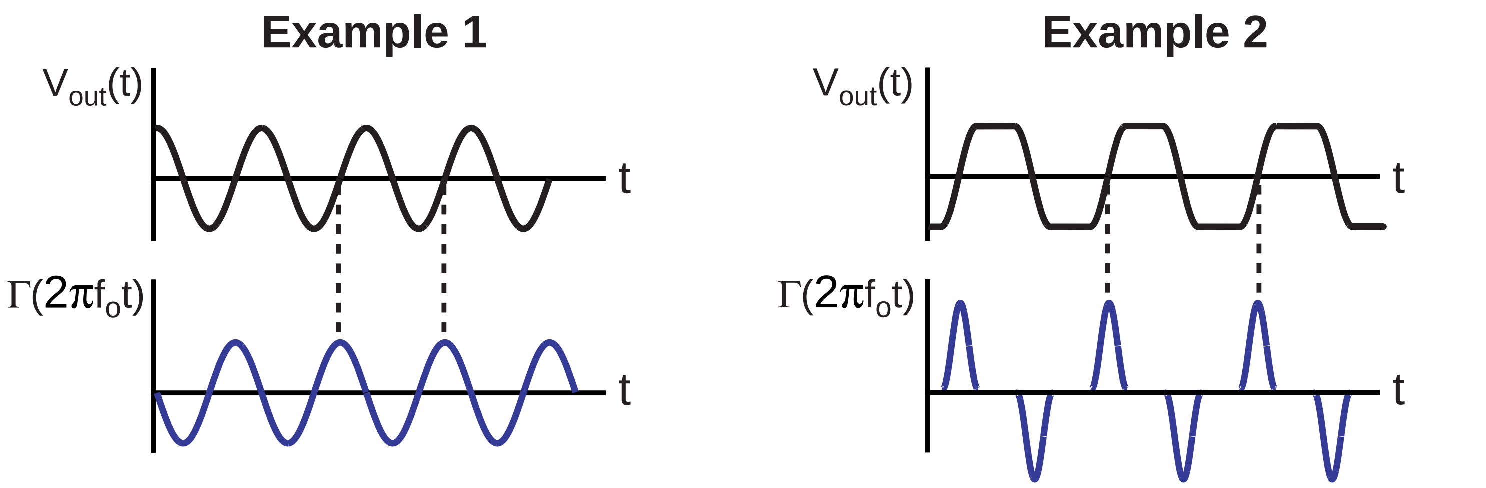
Parameterize Phase Impulse Response in Terms of ISF





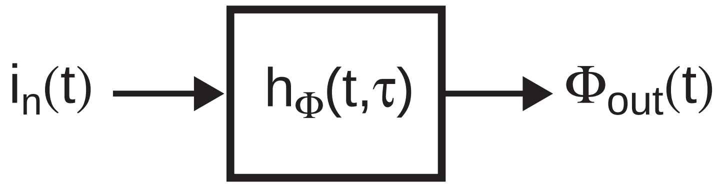
$$h_\Phi(t,\tau) = \frac{\Gamma(2\pi f_o \tau)}{q_{max}} u(t-\tau)$$

Examples of ISF for Different VCO Output Waveforms



- **ISF (i.e., Γ) is approximately proportional to derivative of VCO output waveform**
  - **Its magnitude indicates where VCO waveform is most sensitive to noise current into tank with respect to creating phase noise**
- **ISF is periodic**
- **In practice, derive it from simulation of the VCO**

Phase Noise Analysis Using LTV Framework



- **Computation of phase deviation for an arbitrary noise current input**

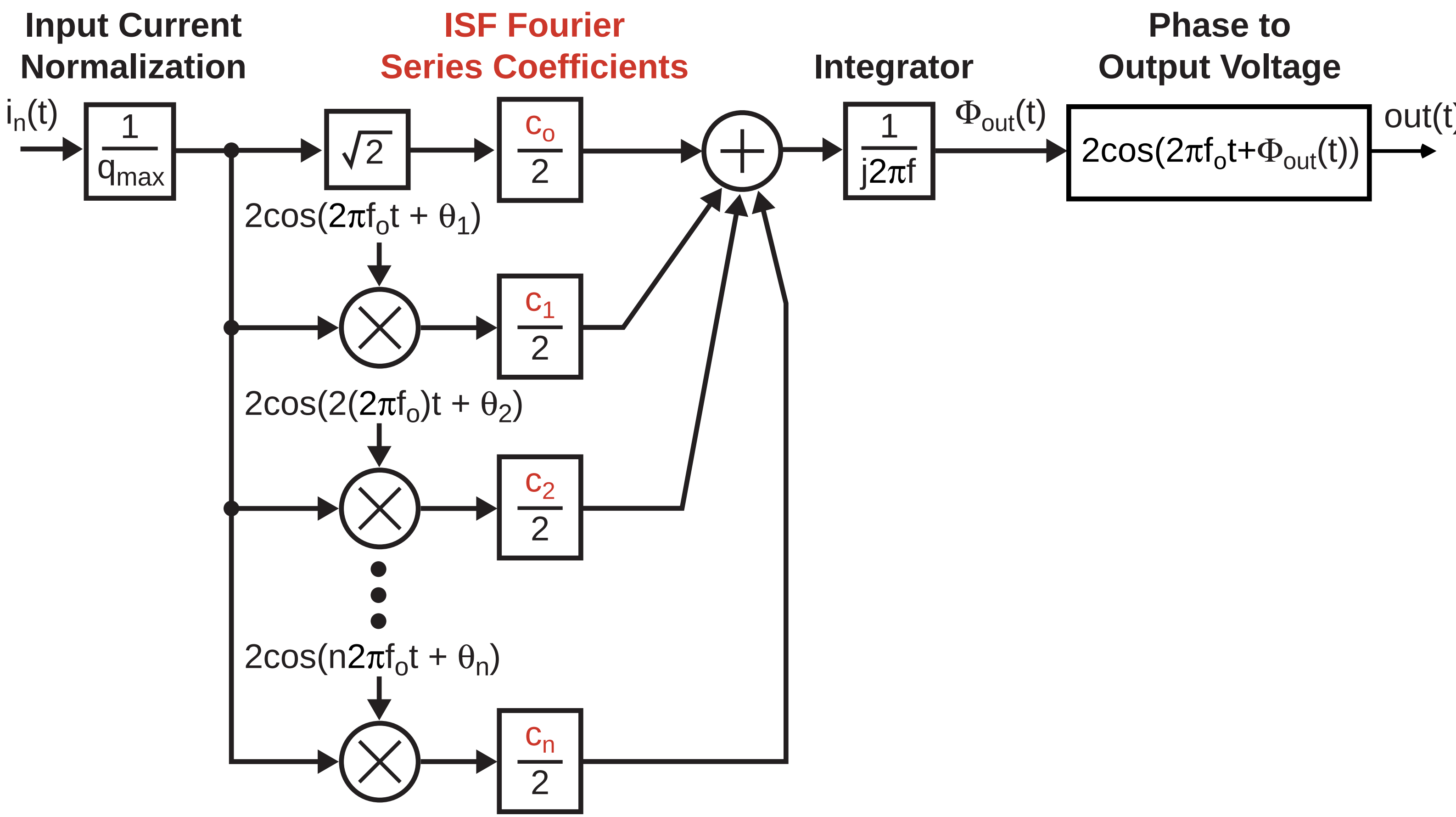
$$\Phi_{out}(t) = \int_{-\infty}^{\infty} h_\Phi(t,\tau) i_n(\tau) d\tau = \frac{1}{q_{max}} \int_{-\infty}^{t} \Gamma(2\pi f_o \tau) i_n(\tau) d\tau$$

- **Analysis simplified if we describe ISF in terms of its Fourier series (note: c_o here is different than book)**

$$\Gamma(2\pi f_o \tau) = \frac{c_o}{\sqrt{2}} + \sum_{n=1}^{\infty} c_n \cos(n 2\pi f_o \tau + \theta_n)$$

$$\Rightarrow \boxed{\Phi_{out}(t) = \int_{-\infty}^{t} \left(\frac{c_o}{\sqrt{2}} + \sum_{n=1}^{\infty} c_n \cos(n 2\pi f_o \tau + \theta_n) \right) \frac{i_n(\tau)}{q_{max}} d\tau}$$

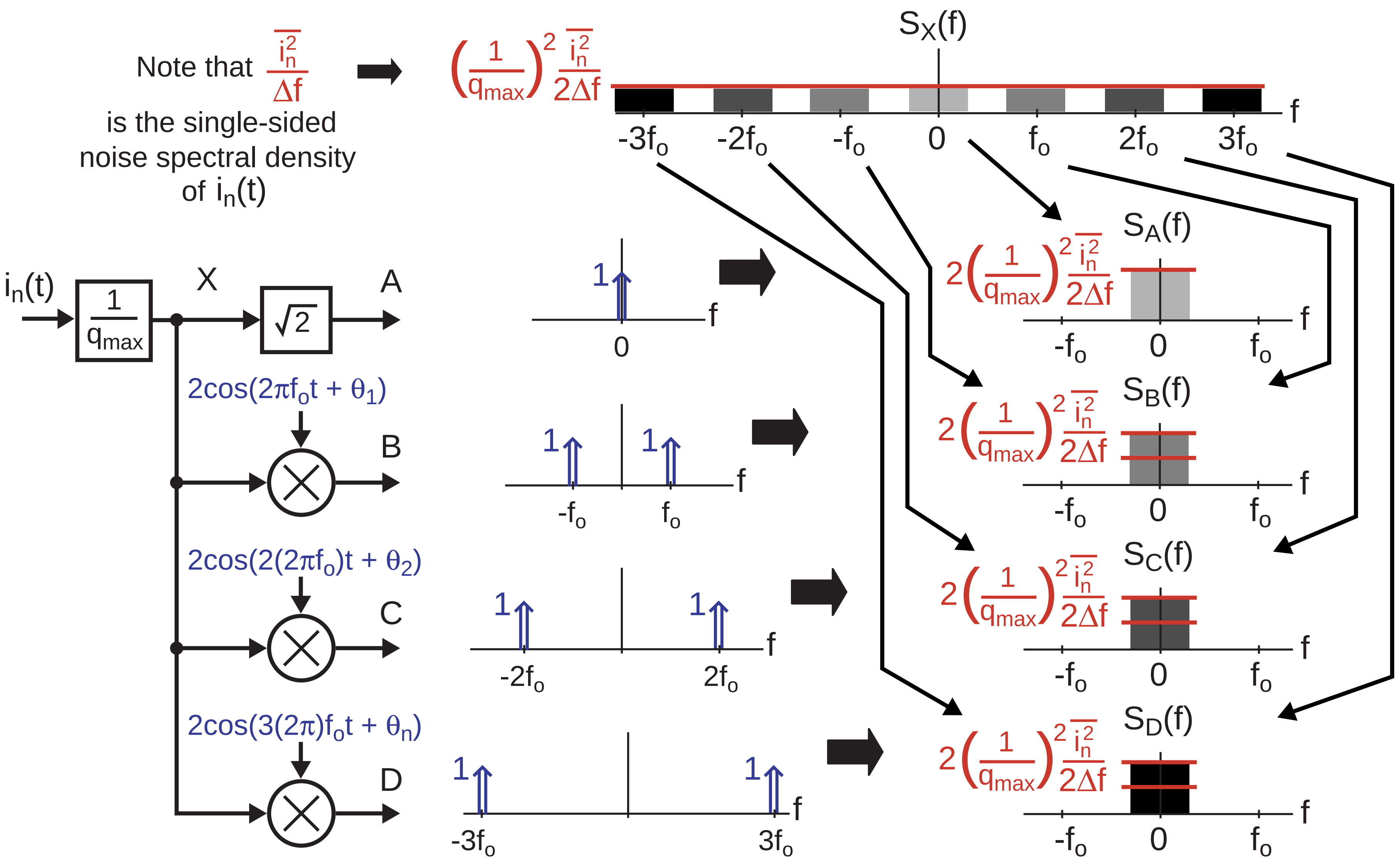
Block Diagram of LTV Phase Noise Expression



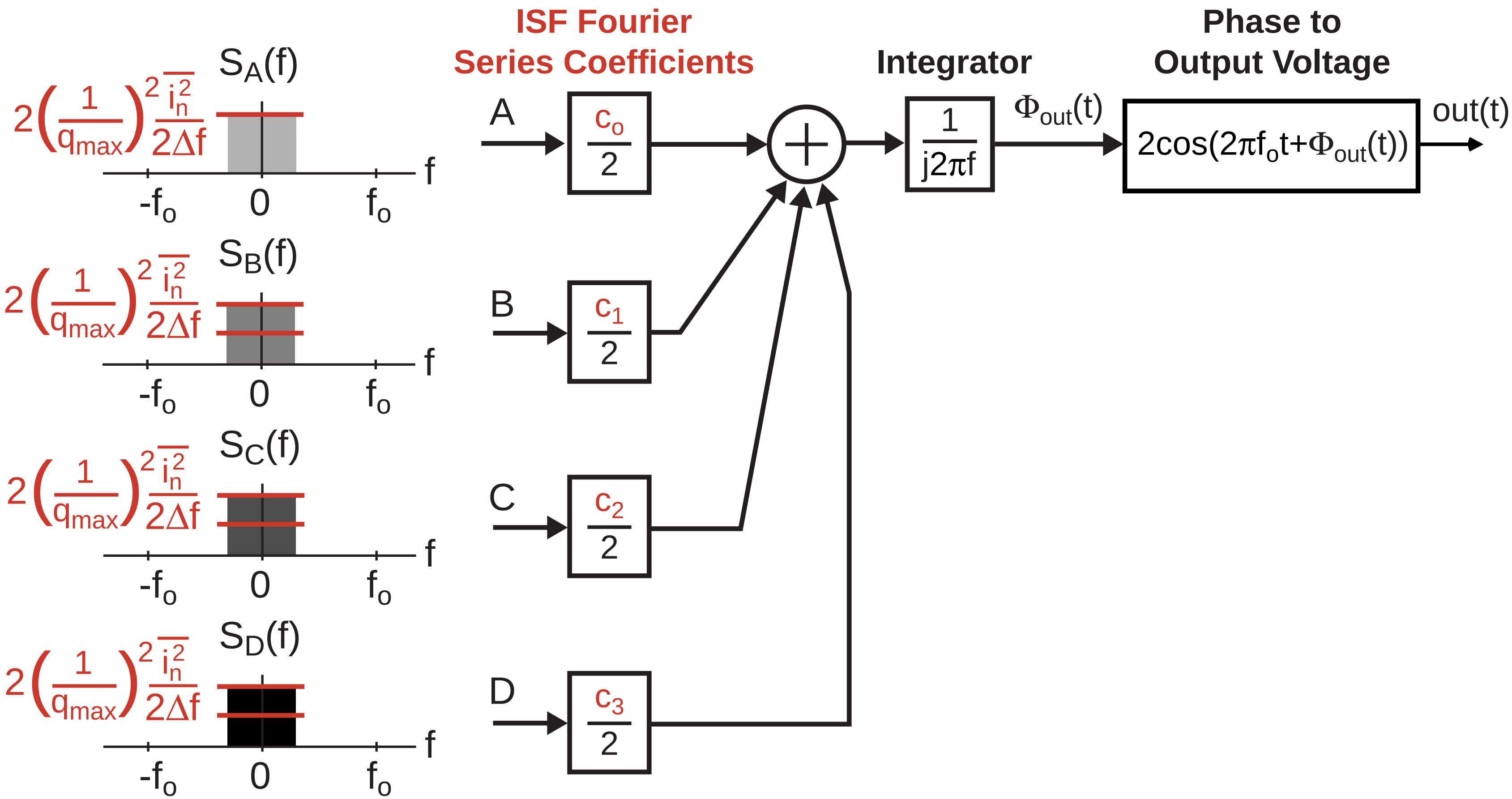
- **Noise from current source is mixed down from different frequency bands and scaled according to ISF coefficients**

Phase Noise Calculation for White Noise Input (Part 1)

Note that $\frac{\overline{i_n^2}}{\Delta f}$ is the single-sided noise spectral density of $i_n(t)$



Phase Noise Calculation for White Noise Input (Part 2)



$$S_{\Phi_{out}}(f) = \left|\frac{1}{j2\pi f}\right|^2 \left(\left(\frac{c_o}{2}\right)^2 S_A(f) + \left(\frac{c_1}{2}\right)^2 S_B(f) + \cdots\right)$$

Spectral Density of Phase Signal

- **From the previous slide**

$$S_{\Phi_{out}}(f) = \left(\frac{1}{2\pi f}\right)^2 \left(\left(\frac{c_o}{2}\right)^2 S_A(f) + \left(\frac{c_1}{2}\right)^2 S_B(f) + \cdots\right)$$

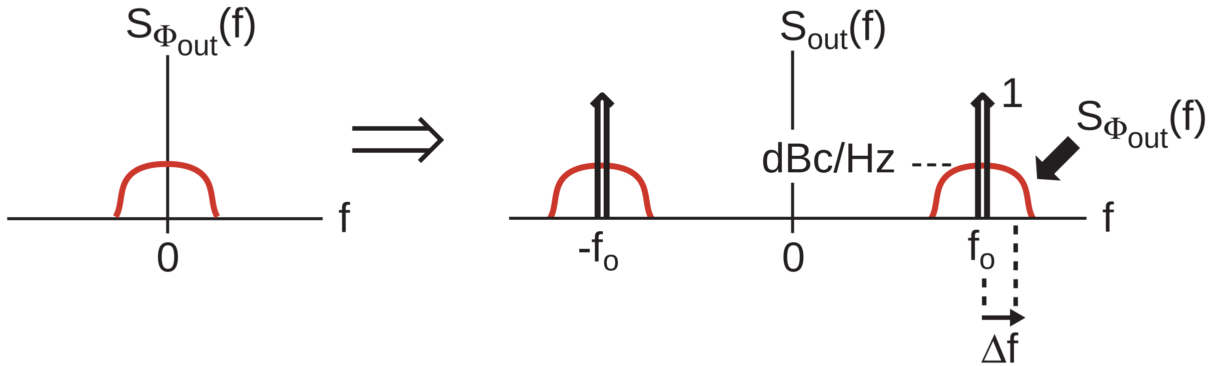
- **Substitute in for $S_A(f)$, $S_B(f)$, etc.**

$$S_{\Phi_{out}}(f) = \left(\frac{1}{2\pi f}\right)^2 \left(\left(\frac{c_o}{2}\right)^2 + \left(\frac{c_1}{2}\right)^2 + \cdots\right) 2 \left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{2\Delta f}$$

- **Resulting expression**

$$S_{\Phi_{out}}(f) = \left(\frac{1}{2\pi f}\right)^2 \left(\sum_{n=0}^{\infty} c_n^2\right) \frac{1}{4} \left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}$$

Output Phase Noise



- **We now know**

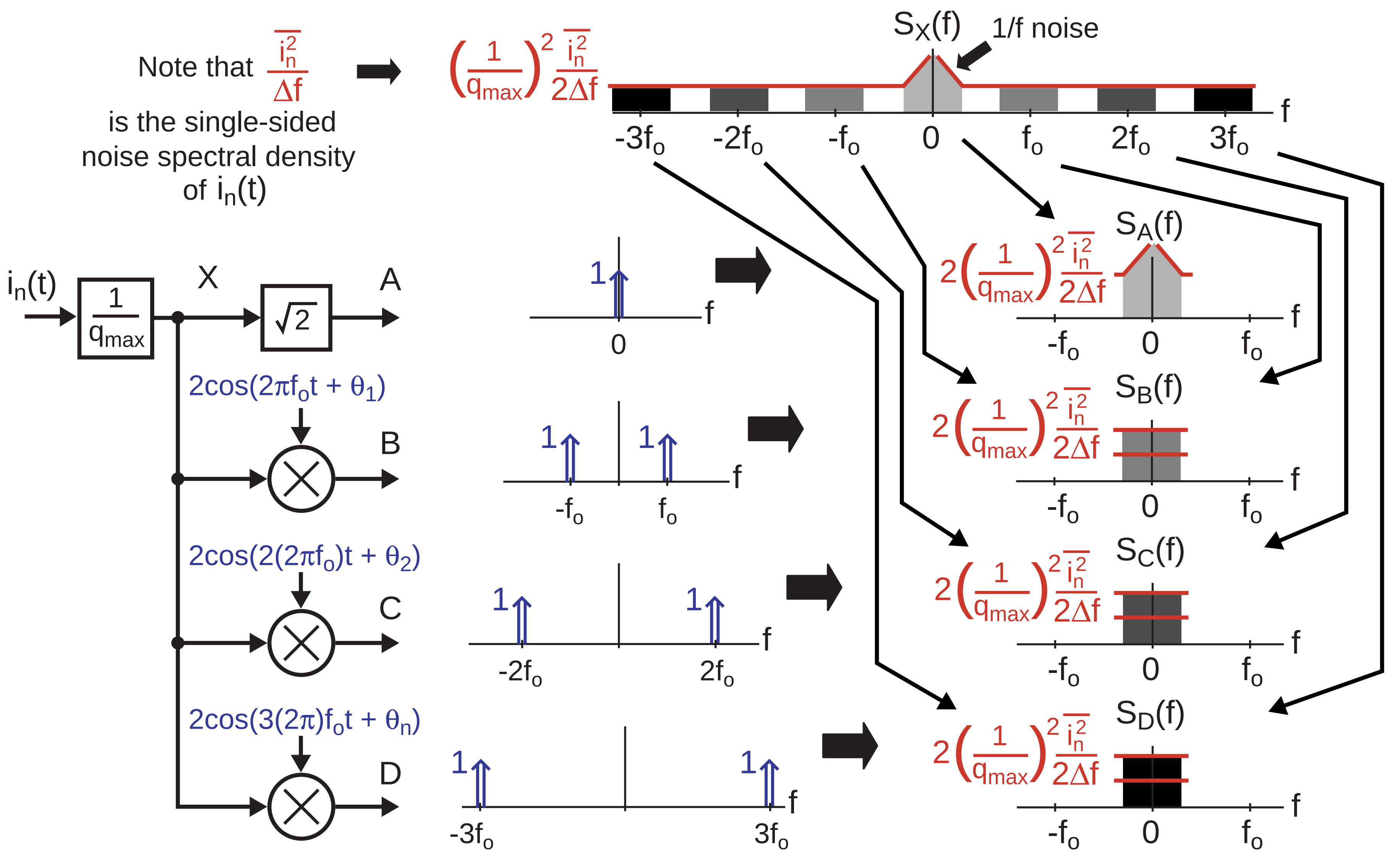
$$S_{\Phi_{out}}(f) = \left|\frac{1}{2\pi f}\right|^2 \left(\sum_{n=0}^{\infty} c_n^2\right) \frac{1}{4} \left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}$$

$$L(\Delta f) = 10\log(S_{\Phi_{out}}(\Delta f))$$

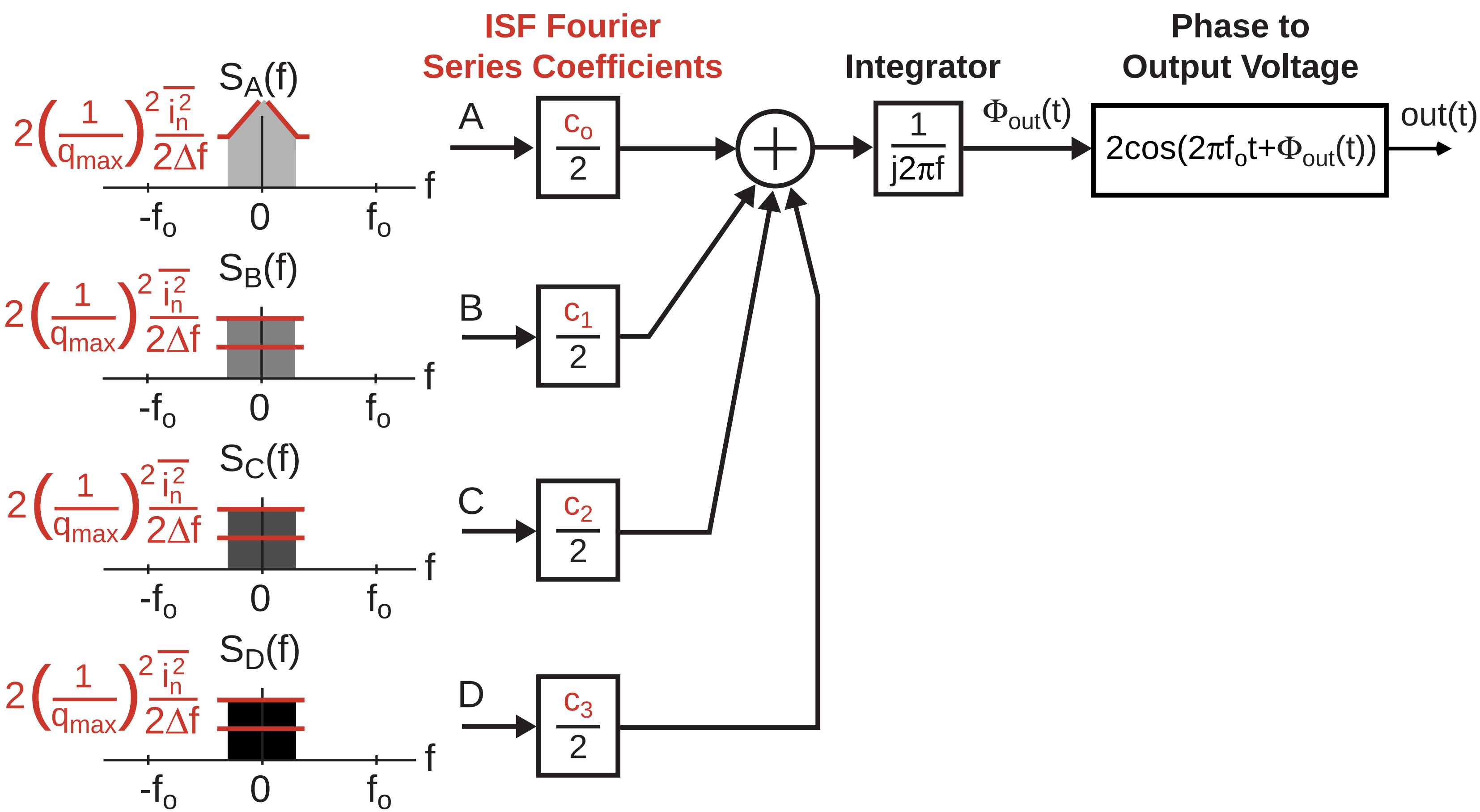
- **Resulting phase noise**

$$L(\Delta f) = 10\log\left(\left(\frac{1}{2\pi\Delta f}\right)^2 \left(\sum_{n=0}^{\infty} c_n^2\right) \frac{1}{4} \left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}\right)$$

The Impact of 1/f Noise in Input Current (Part 1)



The Impact of 1/f Noise in Input Current (Part 2)



$$S_{\Phi_{out}}(f)\Big|_{1/f^3} = \left|\frac{1}{j2\pi f}\right|^2 \left(\frac{c_o}{2}\right)^2 S_A(f)$$

Calculation of Output Phase Noise in 1/f³ region

- **From the previous slide**

$$S_{\Phi_{out}}(f)\Big|_{1/f^3} = \left(\frac{1}{2\pi f}\right)^2 \left(\frac{c_o}{2}\right)^2 S_A(f)$$

- **Assume that input current has 1/f noise with corner frequency $f_{1/f}$**

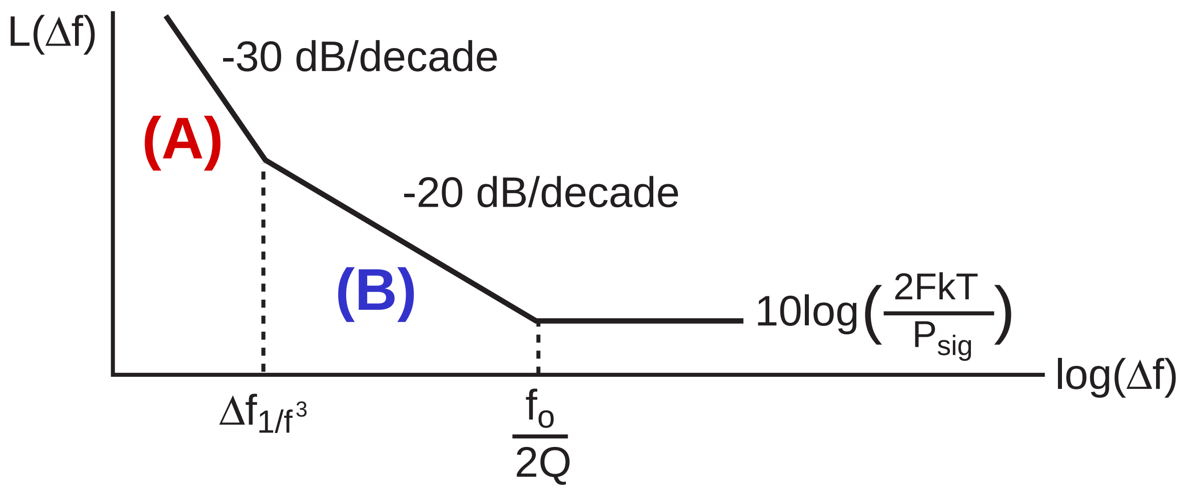
$$S_A(f) = \left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}\left(\frac{f_{1/f}}{\Delta f}\right)$$

- **Corresponding output phase noise**

$$L(\Delta f)\Big|_{1/f^3} = 10\log\left(\left(\frac{1}{2\pi\Delta f}\right)^2 \left(\frac{c_o}{2}\right)^2 S_A(f)\right)$$

$$= 10\log\left(\left(\frac{1}{2\pi\Delta f}\right)^2 \left(c_o^2\right)\frac{1}{4}\left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}\left(\frac{f_{1/f}}{\Delta f}\right)\right)$$

Calculation of $1/f^3$ Corner Frequency



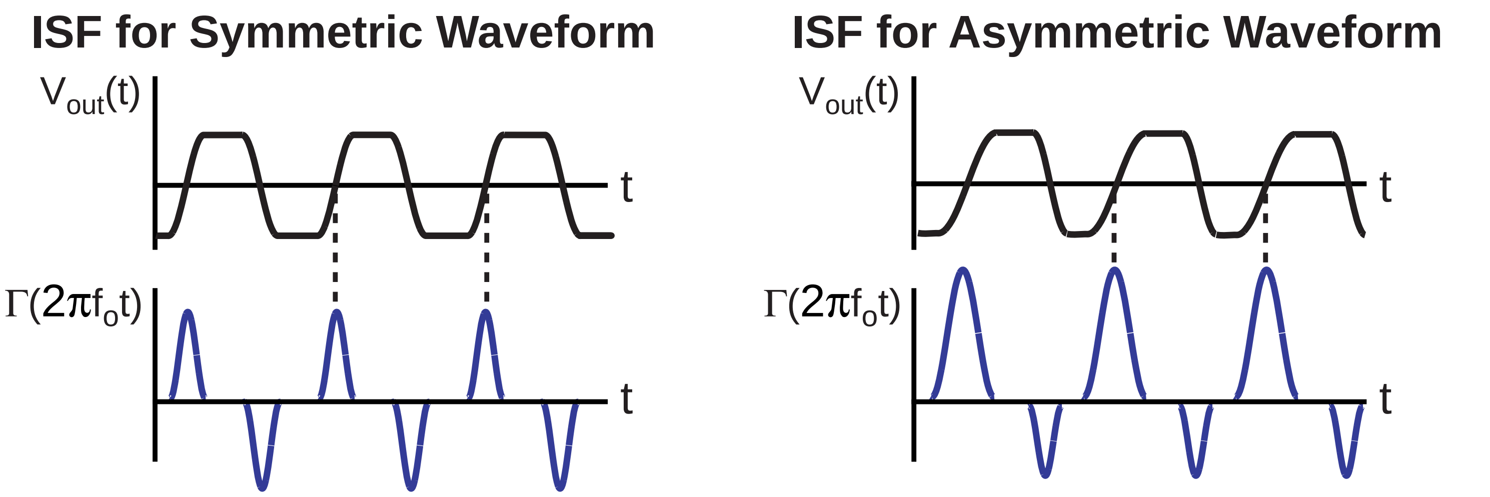
(A) $$L(\Delta f)\Big|_{1/f^3} = 10\log\left(\left(\frac{1}{2\pi\Delta f}\right)^2 (c_o^2)\frac{1}{4}\left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}\left(\frac{f_{1/f}}{\Delta f}\right)\right)$$

(B) $$L(\Delta f) = 10\log\left(\left(\frac{1}{2\pi\Delta f}\right)^2 \left(\sum_{n=0}^{\infty} c_n^2\right)\frac{1}{4}\left(\frac{1}{q_{max}}\right)^2 \frac{\overline{i_n^2}}{\Delta f}\right)$$

(A) = (B) at:

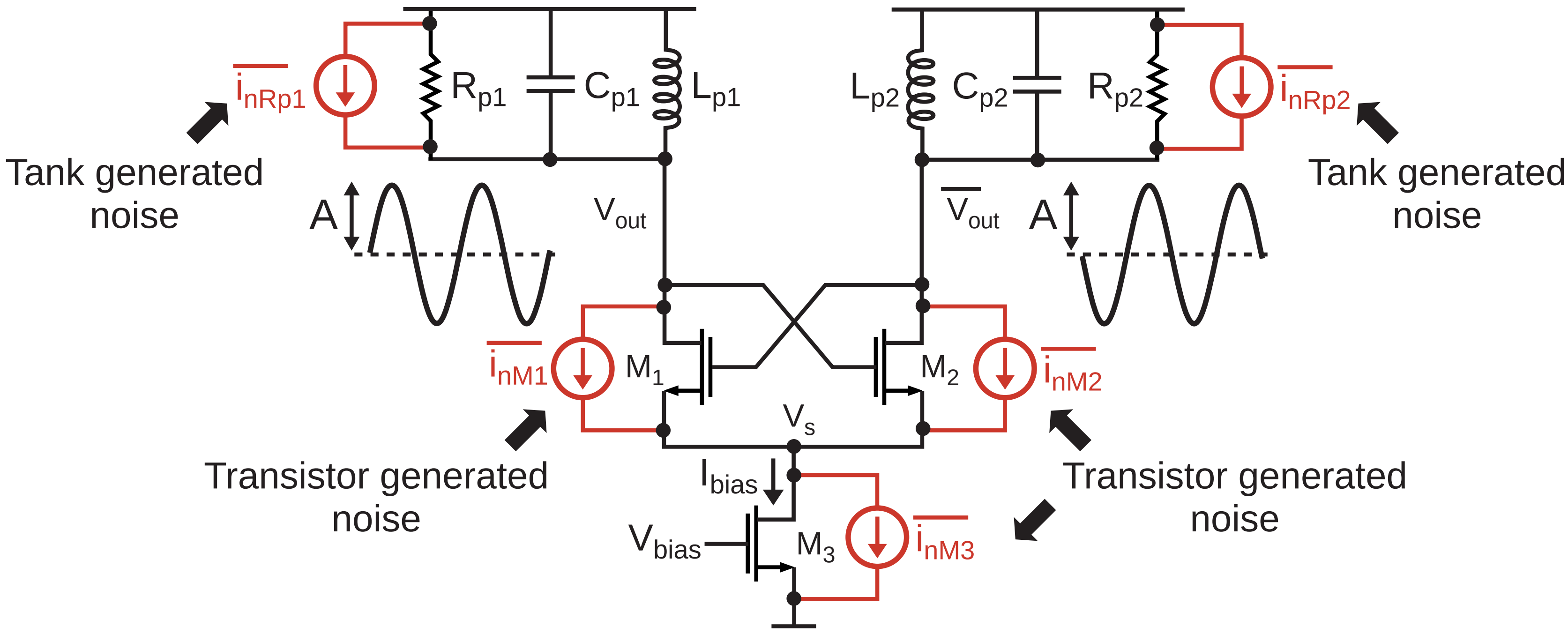
$$\Rightarrow \quad \Delta f_{1/f^3} = \left(c_o^2 \Big/ \sum_{n=0}^{\infty} c_n^2\right) f_{1/f}$$

Impact of Oscillator Waveform on 1/f³ Phase Noise



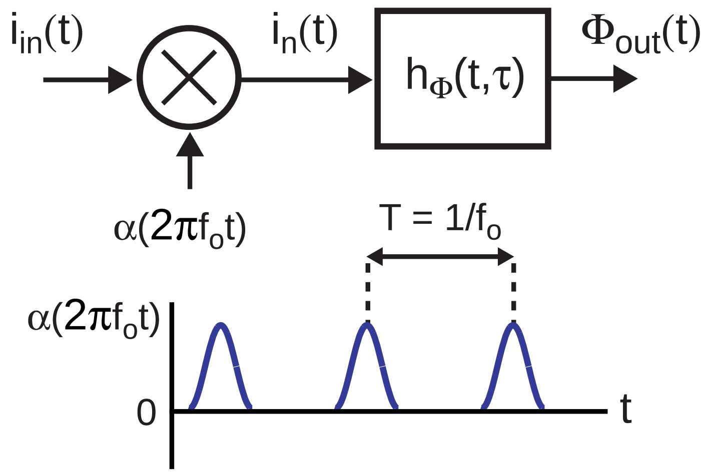
- **Key Fourier series coefficient of ISF for $1/f^3$ noise is c_o**
  - **If DC value of ISF is zero, c_o is also zero**
- **For symmetric oscillator output waveform**
  - **DC value of ISF is zero – no upconversion of flicker noise! (i.e. output phase noise does not have $1/f^3$ region)**
- **For asymmetric oscillator output waveform**
  - **DC value of ISF is nonzero – flicker noise has impact**

Issue – We Have Ignored Modulation of Current Noise



- **In practice, transistor generated noise is modulated by the varying bias conditions of its associated transistor**
  - **As transistor goes from saturation to triode to cutoff, its associated noise changes dramatically**
- **Can we include this issue in the LTV framework?**

Inclusion of Current Noise Modulation



- **Recall**

$$\Phi_{out}(t) = \int_{-\infty}^{\infty} h_\Phi(t,\tau) i_n(\tau) d\tau = \frac{1}{q_{max}} \int_{-\infty}^{t} \Gamma(2\pi f_o \tau) i_n(\tau) d\tau$$

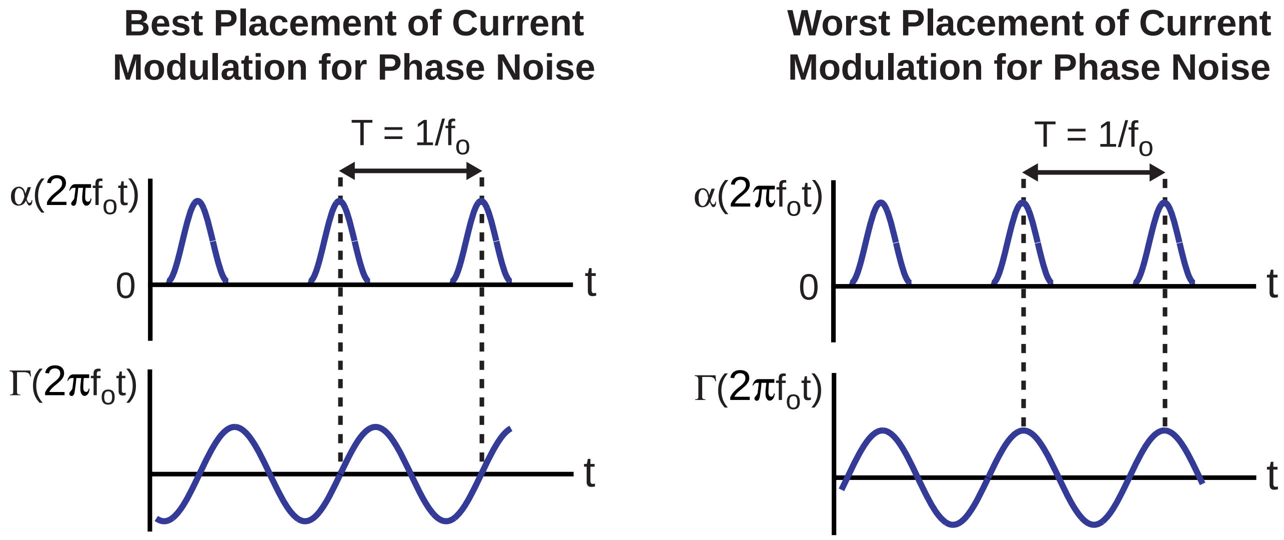
- **By inspection of figure**

$$\Rightarrow \quad \Phi_{out}(t) = \frac{1}{q_{max}} \int_{-\infty}^{t} \underline{\Gamma(2\pi f_o \tau)\alpha(2\pi f_o \tau)} i_{in}(\tau) d\tau$$

- **We therefore apply previous framework with ISF as**

$$\boxed{\Gamma_{eff}(2\pi f_o \tau) = \Gamma(2\pi f_o \tau)\alpha(2\pi f_o \tau)}$$

Placement of Current Modulation for Best Phase Noise

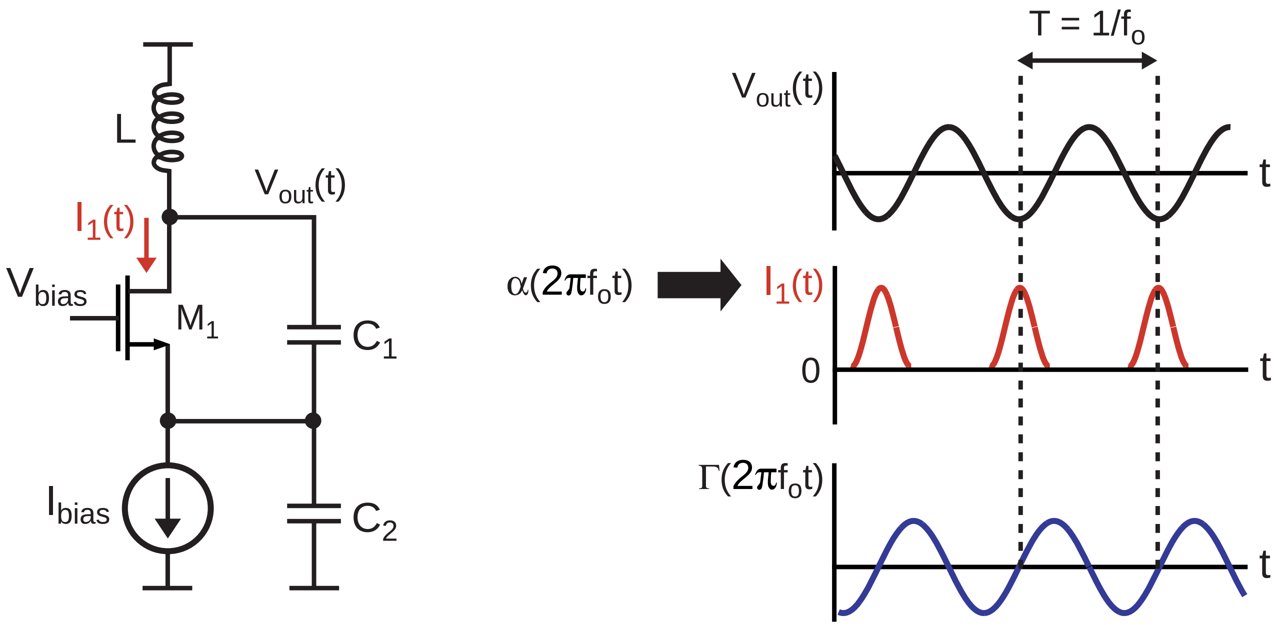


- **Phase noise expression (ignoring 1/f noise)**

$$L(\Delta f) = 10\log\left(\left(\frac{1}{2\pi\Delta f}\right)^2\left(\sum_{n=0}^{\infty} c_n^2\right)\frac{1}{4}\left(\frac{1}{q_{max}}\right)^2\frac{\overline{i_n^2}}{\Delta f}\right)$$

- **Minimum phase noise achieved by minimizing sum of square of Fourier series coefficients (i.e. rms value of Γ_{eff})**

Colpitts Oscillator Provides Optimal Placement of α

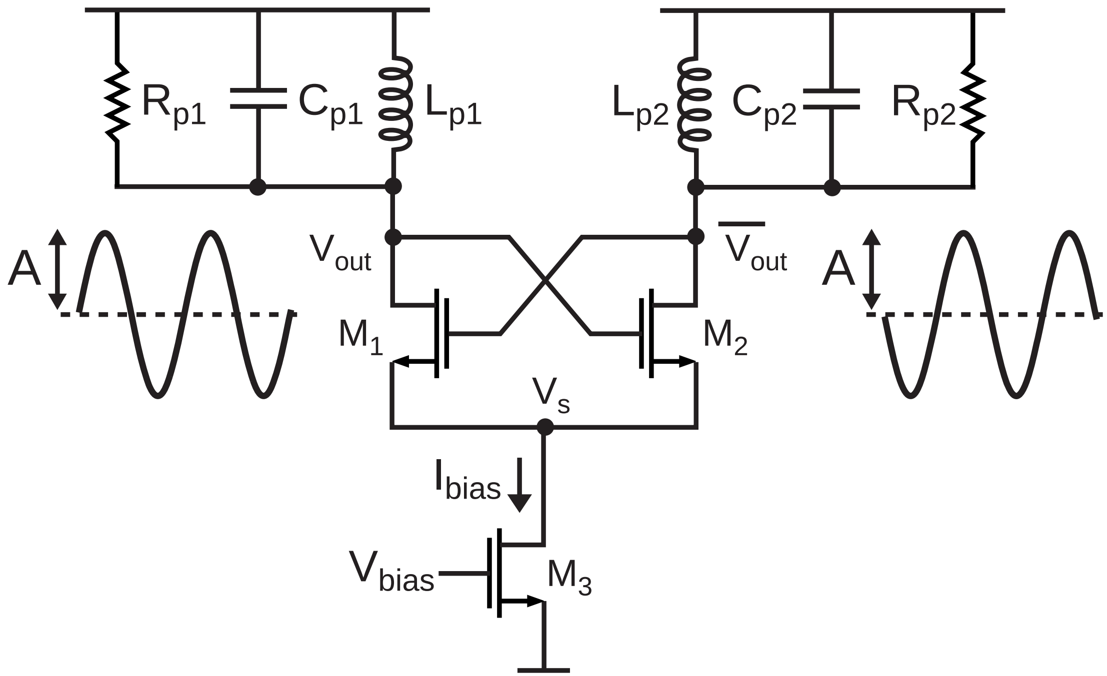


- **Current is injected into tank at bottom portion of VCO swing**
  - **Current noise accompanying current has minimal impact on VCO output phase**

Summary of LTV Phase Noise Analysis Method

- **Step 1: calculate the impulse sensitivity function of each oscillator noise source using a simulator**
- **Step 2: calculate the noise current modulation waveform for each oscillator noise source using a simulator**
- **Step 3: combine above results to obtain $\Gamma_{eff}(2\pi f_o t)$ for each oscillator noise source**
- **Step 4: calculate Fourier series coefficients for each $\Gamma_{eff}(2\pi f_o t)$**
- **Step 5: calculate spectral density of each oscillator noise source (before modulation)**
- **Step 6: calculate overall output phase noise using the results from Step 4 and 5 and the phase noise expressions derived in this lecture (or the book)**

Alternate Approach for Negative Resistance Oscillator

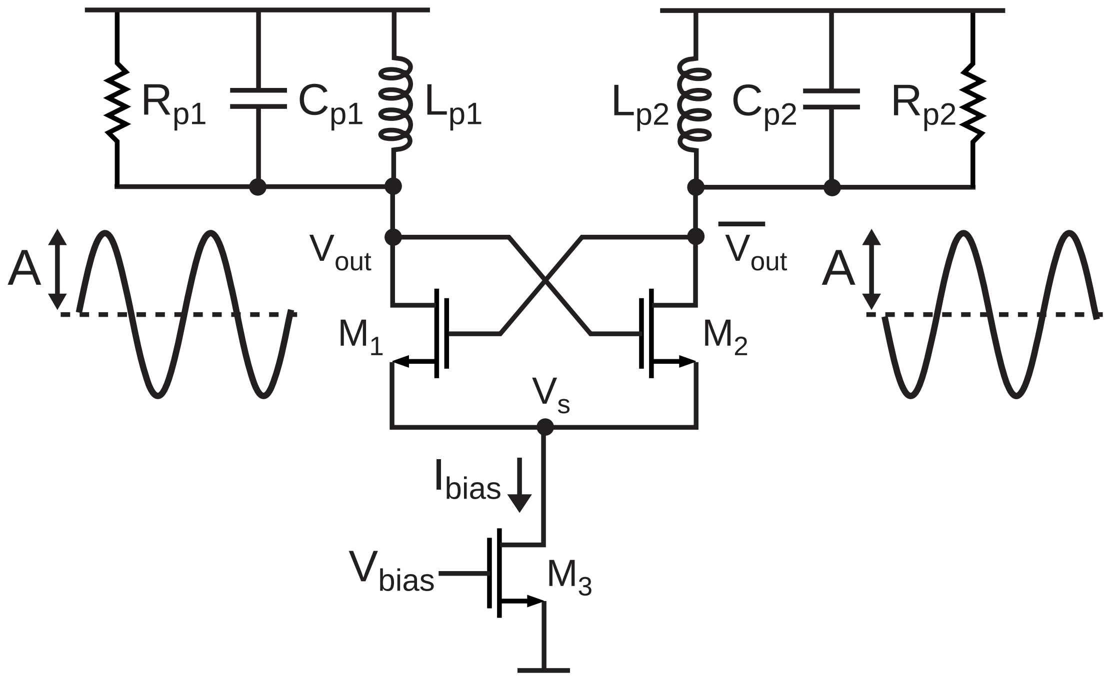


- **Recall Leeson's formula**

$$L(\Delta f) = 10\log\left(\frac{2kTF(\Delta f)}{P_{sig}}\left(\frac{1}{2Q}\frac{f_o}{\Delta f}\right)^2\right)$$

  - **Key question: how do you determine F(Δf)?**

F(Δf) Has Been Determined for This Topology



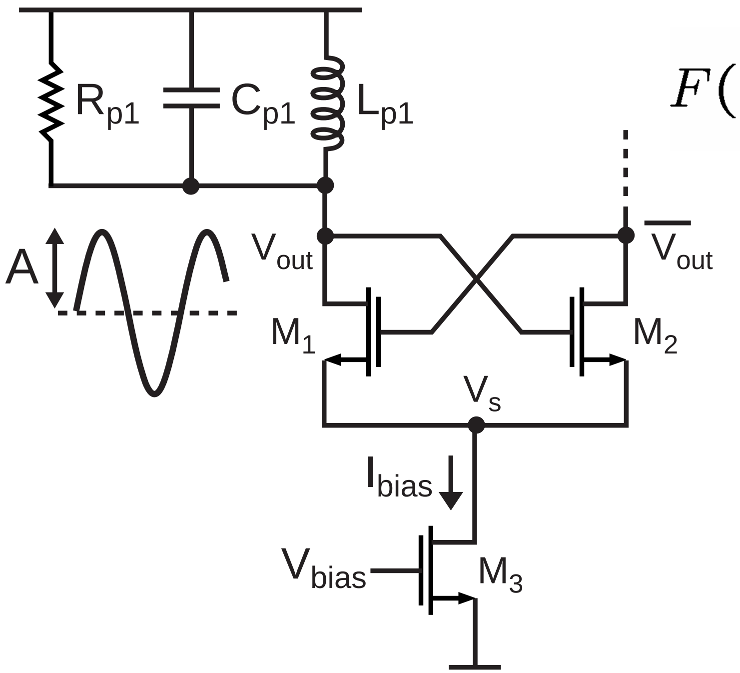
- **Rael et. al. have come up with a closed form expression for F(Δf) for the above topology**
- **In the region where phase noise falls at -20 dB/dec:**

$$F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi A} + \gamma \frac{4}{9} g_{do,M3} R_p \quad (R_p = R_{p1} = R_{p2})$$

References to Rael Work

- **Phase noise analysis**
  - **J.J. Rael and A.A. Abidi, "Physical Processes of Phase Noise in Differential LC Oscillators", Custom Integrated Circuits Conference, 2000, pp 569-572**
- **Implementation**
  - **Emad Hegazi et. al., "A Filtering Technique to Lower LC Oscillator Phase Noise", JSSC, Dec 2001, pp 1921-1930**

Designing for Minimum Phase Noise



$$F(\Delta f) = \underbrace{1}_{(A)} + \underbrace{\frac{2\gamma I_{bias} R_p}{\pi A}}_{(B)} + \underbrace{\gamma \frac{4}{9} g_{do,M3} R_p}_{(C)}$$

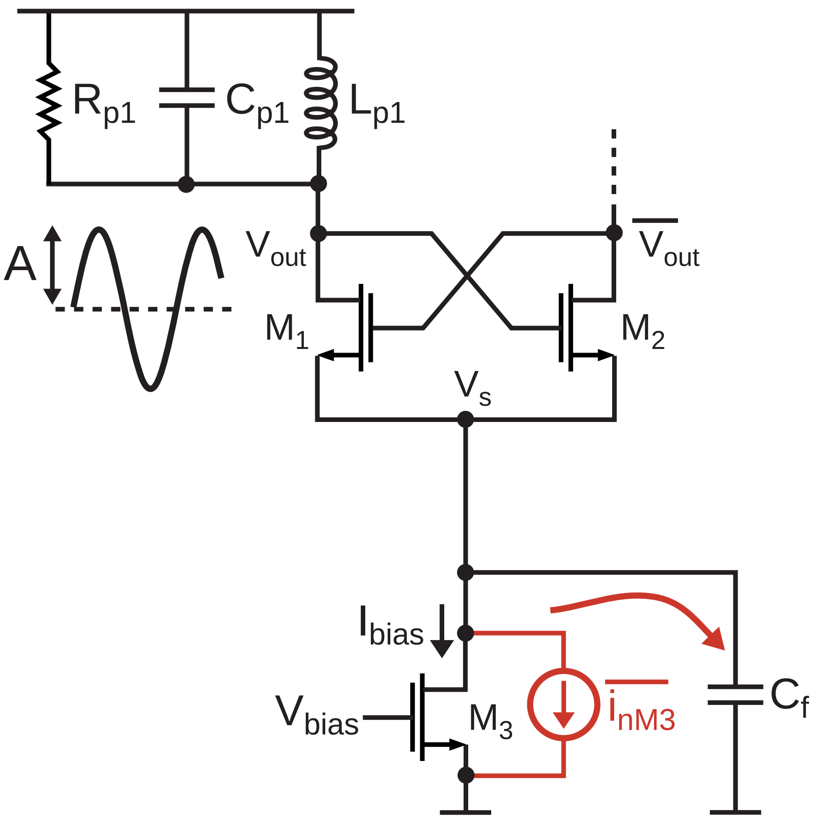
(A) Noise from tank resistance

(B) Noise from M_1 and M_2

(C) Noise from M_3

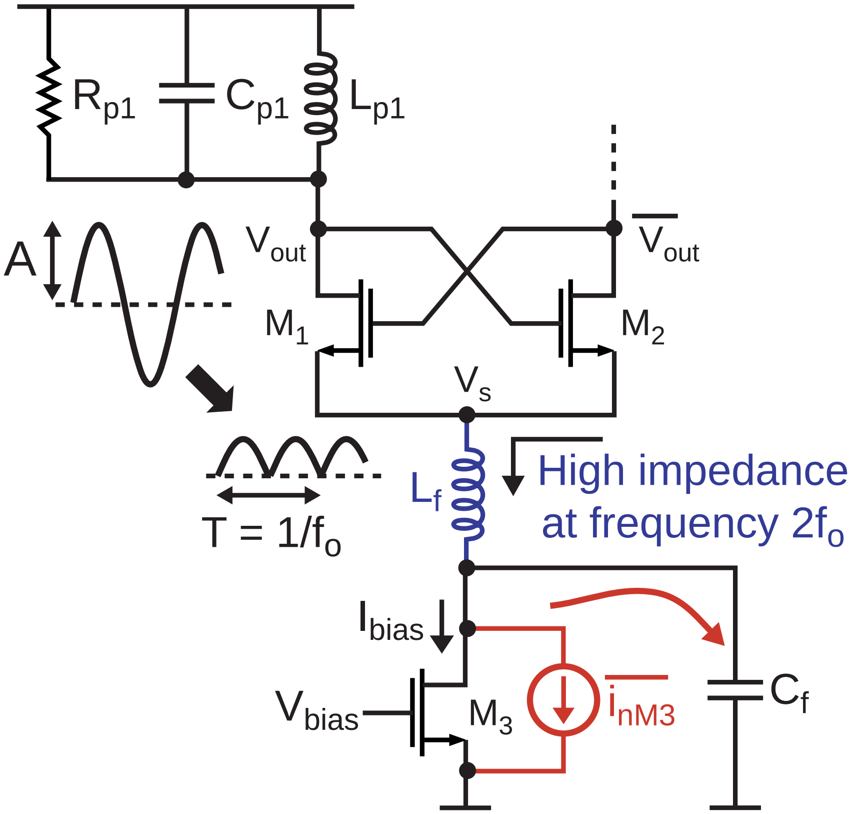
- **To achieve minimum phase noise, we'd like to minimize F(Δf)**
- **The above formulation provides insight of how to do this**
  - **Key observation: (C) is often quite significant**

Elimination of Component (C) in F(Δf)



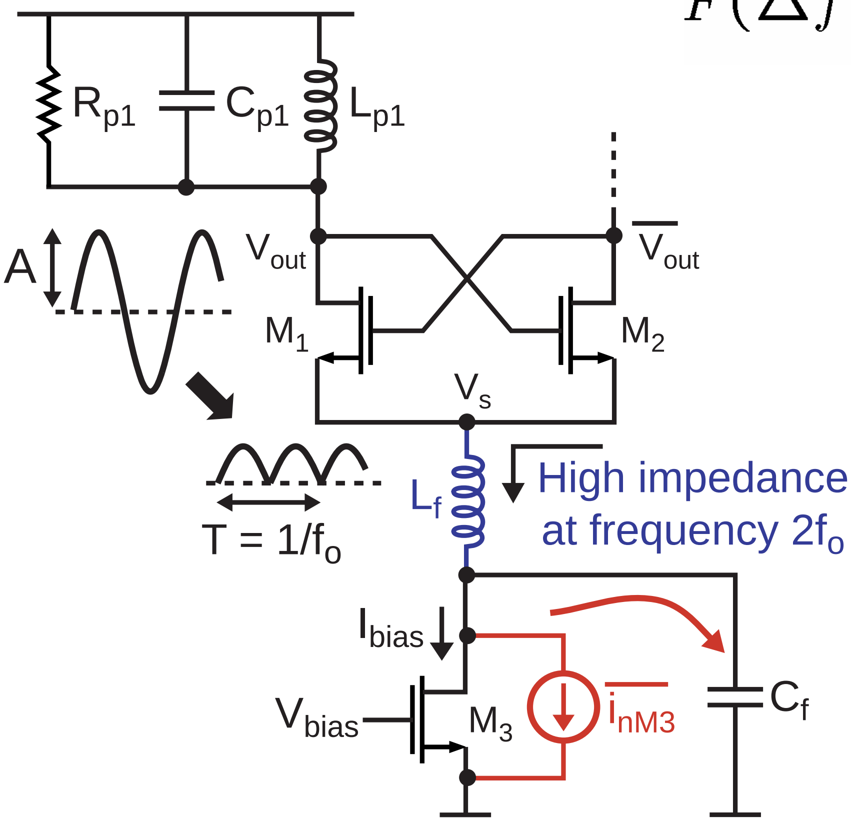
- **Capacitor C_f shunts noise from M_3 away from tank**
  - **Component (C) is eliminated!**
- **Issue – impedance at node V_s is very low**
  - **Causes M_1 and M_2 to present a low impedance to tank during portions of the VCO cycle**
    - Q of tank is degraded

Use Inductor to Increase Impedance at Node V_s



- **Voltage at node V_s is a rectified version of oscillator output**
  - **Fundamental component is at twice the oscillation frequency**
- **Place inductor between V_s and current source**
  - **Choose value to resonate with C_f and parasitic source capacitance at frequency $2f_o$**
- **Impedance of tank not degraded by M_1 and M_2**
  - **Q preserved!**

Designing for Minimum Phase Noise – Next Part



$$F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi A} + \gamma\frac{4}{9} g_{do,M3} R_p$$

(A) (B) (C)

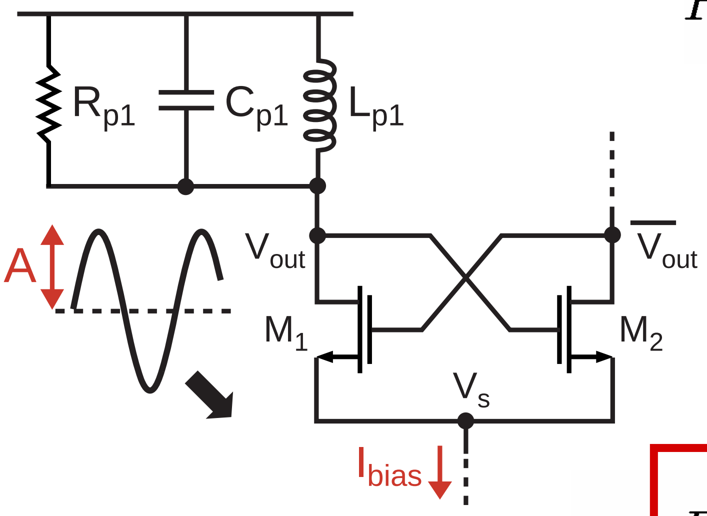
(A) Noise from tank resistance

(B) Noise from M_1 and M_2

(C) Noise from M_3

- **Let's now focus on component (B)**
  - **Depends on bias current and oscillation amplitude**

Minimization of Component (B) in F(Δf)



$$F(\Delta f) = 1 + \underbrace{\frac{2\gamma I_{bias} R_p}{\pi A}}_{(B)}$$

- **Recall from Lecture 11**

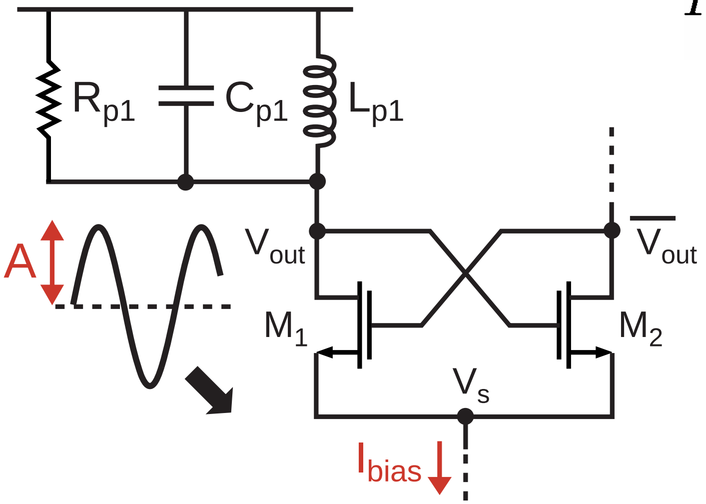
$$A = \frac{2}{\pi} I_{bias} R_p$$

$$\Rightarrow \boxed{F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi (2/\pi) I_{bias} R_p} = 1 + \gamma}$$

- **So, it would seem that I_{bias} has no effect!**
  - **Not true – want to maximize A (i.e. P_{sig}) to get best phase noise, as seen by:**

$$L(\Delta f) = 10 \log \left(\frac{2kTF(\Delta f)}{P_{sig}} \left(\frac{1}{2Q} \frac{f_o}{\Delta f} \right)^2 \right)$$

Current-Limited Versus Voltage-Limited Regimes



$$F(\Delta f) = 1 + \frac{2\gamma I_{bias} R_p}{\pi A}$$

(B)

- **Oscillation amplitude, A, cannot be increased above supply imposed limits**
- **If I_{bias} is increased above the point that A saturates, then (B) increases**

- **Current-limited regime: amplitude given by** $A = \frac{2}{\pi} I_{bias} R_p$
- **Voltage-limited regime: amplitude saturated**

Best phase noise achieved at boundary between these regimes!

Summary

- **Leeson's model is outcome of linearized VCO noise analysis**
- **Hajimiri method provides insights into cyclostationary behavior, 1/f noise upconversion and impact of noise current modulation**
- **Rael method useful for CMOS negative-resistance topology**
  - **Closed form solution of phase noise!**
  - **Provides a great deal of design insight**
- **Practical VCO phase noise analysis is done through simulation these days**
  - **Spectre RF from Cadence, FastSpice from Berkeley Design Automation is often utilized to estimate phase noise for integrated oscillators**